電機所

1. (a) Show that the homogeneous Cauchy-Euler equation

$$Ax^2\frac{d^2y}{dx^2}+Bx\frac{dy}{dx}+Cy=0$$

can be transformed to the constant coefficient equation shown below. (5%)

$$Ay''+(B-A)y'+C=0$$

(b) Apply the result, if needed, to solve the following differential equation. (10%)

$$(4x^2+12x+9)y''+(12x+18)y'+4y=0$$

2. The Dirac delta function is defined as $\delta(t)=\lim_{\varepsilon\to 0^+}\delta_\varepsilon(t)$ with the function $\delta_\varepsilon(t)$ of the form

$$\delta_\varepsilon(t)=\begin{cases}\frac{1}{\varepsilon} & \text{if } 0\le t<\varepsilon \\ 0 & \text{if } t<0 \text{ or } t\ge\varepsilon\end{cases}$$

(a) Please evaluate the convolution $\delta(t-a)*f(t)$. (10%) where $f(t)$ is any given function.

(b) Use Laplace transform to solve $y'+5y=\delta'(t)$ with $y(0)=0$. (10%)

3. A vector $\vec{F}=xy^2\,\hat{i}+(x^2y+y^3)\,\hat{j}$, and

a curve C is defined as $\vec{r}(t)=\cos(t)\,\hat{i}+\sin(t)\,\hat{j}+t\,\hat{k},\ 0\le t\le\frac{3}{2}\pi$

(1) find the line equation of this curve and draw the curve. (5%)

(2) find the length of this curve. (5%)

(3) Evaluate the vector integral (5%)

$$\int_C \vec{F}\bullet d\vec{r}$$

4. For the differential equation $(x^2-x)y''-xy'+y=0$,

(1) find the recurrence equation. (4%)

(2) find one solution. (3%)

(3) utilize the solved solution above to find the other solution with the method of variation of parameters. (3%).

5. Solve the partial differential equation with Laplace and Fourier transform.(10%)

$$\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}\quad(-\infty<x<\infty,\ t>0)$$

$$u(x,0)=2e^{-2|x|}\quad(-\infty<x<\infty)$$

$$\frac{\partial u}{\partial t}(x,0)=0\quad(-\infty<x<\infty)$$

(To Be Continued)

6. (a) Obtain the Fourier series representation of the function f(x),

$$f(x+4) = f(x) \text{ and } f(x)=\begin{cases} x+2, & \text{for } -2 < x < 0 \\ 1, & \text{for } 0 < x < 2 \end{cases} \qquad (10\%)$$

(b) Calculate $\sum_{m=1}^{\infty} \frac{1}{(2m-1)^2}$ (4%)

7. Compute $\int_C f(z)\, dz$

in each case if f(z)=(z+2)/z and C is as shown

(a) C: $x^2 + y^2 = 4$ from (2 , 0) to (-2 , 0); counterclockwise (8%)

(b) C: $x^2 + y^2 = 4$ closed path; clockwise (8%)