1. (a) A system has a characteristic equation $s^6+s^5+6s^4+6s^3+11s^2+8s+12=0$. Utilizing the Routh-Hurwitz stability criterion, determine the number of roots, if any, in the right-half s-plane and on the $j\omega$-axis, respectively. (7%)
   (b) A system has a characteristic equation $s^3+\alpha s^2+(2+\beta)s+(1+\beta)=0$. Utilizing the Routh-Hurwitz stability criterion, find the values of α and β which will cause sustained oscillations in this system at a frequency 5 rad/sec. (10%)

2. Consider the temperature control system shown in Fig. 1.
   (a) Find the pulse transfer function C(z)/R(z) with T=2 s and D(z)=1. (4%)
   (b) Evaluate the system response c(k) for a unit-step input. (4%)
   (c) Find the dc gain of this system and the steady-state output of (b). (4%)
   (d) Find the time constant τ for the transient response of this system. (4%)

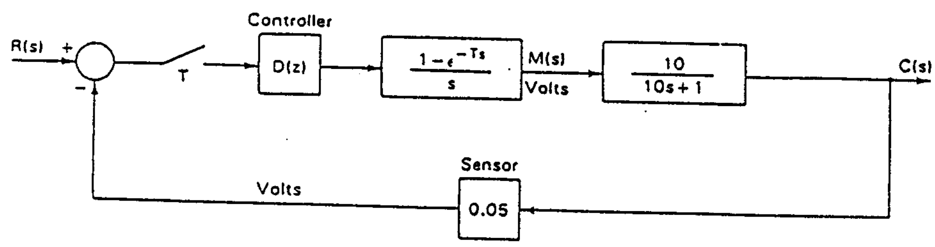

Fig. 1

3. (a) Draw the root locus of the system shown in Fig. 2. (10%)
   (b) Draw the Bode Plot of the system shown in Fig. 2 for K=0.1, 0.2 and 10, respectively. (11%)
   (c) Based on (b), show why the gain margin and phase margin of a minimal phase open-loop system with a negative unit feedback are shown as Fig. 3. (13%)

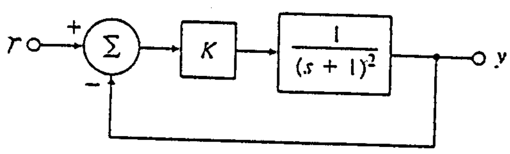

Fig. 2

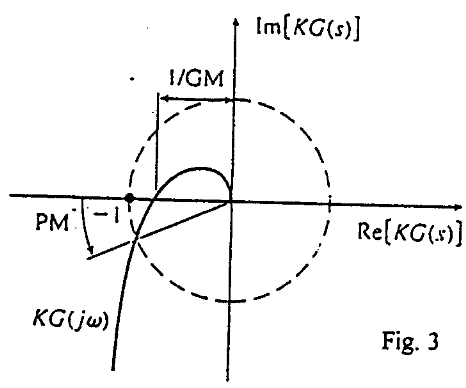

Fig. 3

4. Please define addition operator "+" and multiplication operator "·" such that the set {0, α, 1} forms a field where $\alpha \neq 0$ and $\alpha \neq 1$. (8%)

5. Let E_1 and E_2 be $q \times p$ and $p \times q$ constant matrices, respectively, please show that $E_1(I_{p\times p}+E_2E_1)^{-1} = (I_{q\times q}+E_1E_2)^{-1}E_1$ if $\det(I_{q\times q}+E_1E_2)\neq 0$. (7%)

6. If the state transition matrix of $\dot{X}(t)=A(t)X(t)$ is $\Phi(t,t_o)=\begin{bmatrix} e^{-(t-t_o)} & (e^{t+t_o}-e^{-(t-3t_o)})/2 \\ 0 & e^{-(t-t_o)} \end{bmatrix}$, please find $A(t)$. (10%)

7. Given the z-transform of $\{e(k)\}$ is $E(z)$. Please show that $\lim_{n\to\infty} e(n) = \lim_{z\to 1}(z-1)E(z)$ provided that the left-side limit exists. (8%)