(10%) 1. In how many ways can a gambler draw five cards from a standard deck and get (a) three of a kind (b) four of a kind ?

(10%) 2. For $n \in Z^+$, let $U = \{1, 2, 3, \ldots, n\}$. Define the relation R on $P(U)$: the power set of U by ARB if $A \nsubseteq B$ and $B \nsubseteq A$. How many ordered pairs are there in this relation?

(10%) 3. Let $A = \{a, b, c, d, e, f, g\}$. How many of relations on A are there such that they all are equivalence relations with $b \in [c]$, $d \in [e]$, and $[c] \neq [d]$?

(10%) 4. Find the value of $3 \cdot 2 \cdot 1 + 4 \cdot 3 \cdot 2 + \cdots + (n+1)n(n-1)$.
(hint: use the generating function)

(10%) 5. Five professor named A, B, C, E, and D are to be assigned to teach one class each from among CAD-I, CAD-II, CAD-III, VLSI, and IC Design. Professor A will not teach CAD-II or IC Design, C can not stand VLSI, B and E both refuse to teach CAD-I or CAD-III, and D detests CAD-II. What is the probability that C or E will get to teach CAD-II.

(10%) 6 Solve the recurrence equation

$$a_{n+2} - a_n = \sin(n\pi/2), \quad n \geq 0, \; a_0 = 1, \; a_1 = 1$$

(10%) 7. If color the edges of complete graph K_6 using two colors: red and green randomly, please prove that there always exists a complete subgraph K_3 with the same edge-color in it.

(10%) 8. There are seven people A, B, C, D, E, F, and G to take a dinner in a room. How to arrange them to sit around a round table such that they can talk each other with their neighbors using the same language? If A only can speak English; B can speak both English and Germanic; C can speak English, Italian, and Russian; D can speak Chinese and Japanese; E can speak Italian and Germanic; F speaks Russian, Japanese, and French; and G can speak Germanic and French.

(10%) 9. Let $A=R^+$. Define $\oplus$ and $\odot$ by $a \oplus b = a * b$, and $a \odot b = a^{\log_2 b}$, $a, b \in A$. a) Is $(A, \oplus, \odot)$ is a commutative ring? (Give your reason)

b). Is it an integral domain or field? (Give your reason).

(10%) 10. A pyramid has a square base and 4 faces that are equilaterial triangles. If we can move the pyramid in space, how many nonequivalent ways are there to paint its 5 faces if we have paint of 4 different colors?