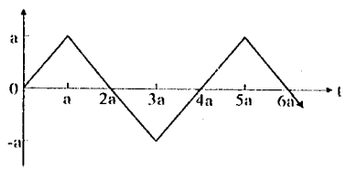
1. Solve for the following differential equations or initial value problems.

(a) $ye^{xy}\frac{dx}{dy}+xe^{xy}=12y^2,\quad y(0)=-1.$ **(5%)**

(b) $4y''+36y=\csc 3x.$ **(5%)**

(c) $y=xy'+\frac{1}{2}(y')^2.$ **(5%)**

2. Please find the Laplace transform of the periodic function with period of 4a shown below. **(10%)**



3. A vector $\vec{F} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$, and S is the closed cone surface. The closed cone surface equation is $z = (x^2 + y^2)^{1/2}$, $0 \le z \le 1$ (including the cap surface equation, $x^2 + y^2 = 1$, $z=1$). $\hat{i}$, $\hat{j}$, $\hat{k}$ are three orthonormal unit vectors.

(a) Evaluate the vector integral with this two surfaces (cone surface and cap surface) **(8%)**

$$\oiint_S \vec{F}\bullet d\vec{A}$$

(b) Using the Divergence theorem of Gauss to re-evaluate the vector integral shown above. **(7%)**

4. Consider the waveform of vibrations in a circular membrane. The partial differential equation is shown as follows:

$$\frac{\partial^2 z}{\partial t^2}=4\nabla^2 z=4\left[\frac{\partial^2 z}{\partial r^2}+\frac{1}{r}\frac{\partial z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 z}{\partial\theta^2}\right](0\le r\le R,-\pi\le\theta\le\pi)$$

The preset conditions are:

$$z(r,-\pi,t)=z(r,\pi,t)$$
$$\frac{\partial z}{\partial\theta}(r,-\pi,t)=\frac{\partial z}{\partial\theta}(r,\pi,t)$$
$$z(r,\theta,0)=f(r,\theta)$$
$$\frac{\partial z}{\partial t}(r,\theta,0)=0$$
$$z(R,\theta,t)=0$$

Please find the general solution using the separation of variables method (You don't need to solve the coeffcients). **(10%)**

(背面仍有題目,請繼續作答)

5. $f(t)$ and $g(t)$ are given as follows:

$$f(t)=\begin{cases}1 & -2<t<2\\ 0 & otherwise\end{cases} \quad \text{and} \quad g(t)=\begin{cases}1 & 0<t<2\\ 0 & otherwise\end{cases}$$

Let $h(t)$ is the convolution of $f(t)$ and $g(t)$. $h(t)=f(t)*g(t)$

(a) Find and draw the function of $h(t)$ **(5%)**

(b) Determine the Fourier transform of $h(t)$ **(5%)**

6. Determine the value of the following integrals **(5%)**

$$\int_0^\infty \left(\frac{\sin 5t}{t}\right)^2 dt$$

7. Find the eigenvalue and eigenfunction of the following Strum-Liouville problem. **(10%)**

$$y''+\lambda y=0; \quad y(0)=y'(\pi)=0$$

8. Assume $k>k'>0$, calculate **(10%)**

$$\int_0^\infty \frac{\sin kx \sin k'x}{x^2+a^2}\,dx$$

9. (a) Calculate for real a, $0<a<1$ **(10%)**

$$\int_0^\infty \frac{x^{2a-1}}{1+x}\,dx$$

(b) Is it necessary to have a restriction on a ? Why? **(5%)**