1. A vector with nonegative entries that add up to one is called a probability vector. Define a stochastic matrix be a square matrix whose columns are probability vector. Given a stochastic matrix

$$P=\begin{bmatrix}0.5 & 0.2 & 0.3\\ 0.3 & 0.8 & 0.3\\ 0.2 & 0 & 0.4\end{bmatrix},$$

find a probability vector q such that Pq=q. (10%)

2. The set of all vectors that are orthogonal to a space W is called the orthogonal complement of W. Let A be an $m\times n$ matrix. Prove that the orthogonal complement of the row space of A is the null space of A. (7%)

3. A matrix A is said to be orthogonally diagonalizable if there is an orthogonal matrix P (i.e., $P^{-1}=P^T$) and a diagonal matrix D such that $A=PDP^T$.
(a) Prove that if A is orthogonally diagonalizable then A is a symmetrix matrix. (5%)
(b) Let $A=\begin{bmatrix}4 & 2\\ 2 & 1\end{bmatrix}$, orthogonally diagonalize the matrix A. (8%)

4. Let f(t), g(t), and h(t) be defined as shown in Fig.1.

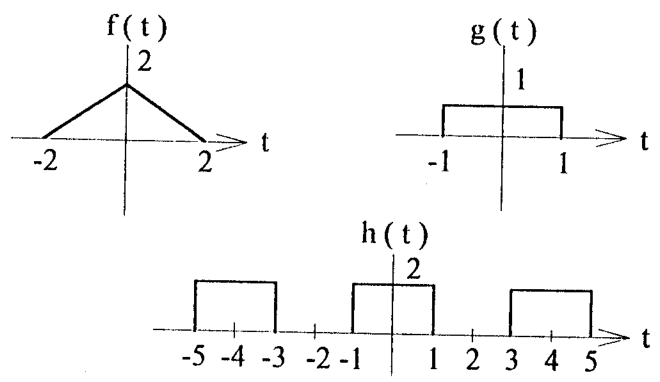

Fig.1

(a) Find the Fourier transform of f(t). (6%)
(b) Find the convolution of f(t) and g(t). (6%)
(c) h(t) is a periodic function. Find and sketch the autocorrelation function of h(t). (8%)

5. The number of buses arriving at a bus station is a Poisson random variable with mean=10 buses/hour. Note that the general probability mass function (pmf) of the Poisson random variable is $P(k)=\frac{\lambda^k e^{-\lambda}}{k!}$.

(a) What is the probability that no car will arrive in any half-hour period? (5%)

(b) What is the probability density function (pdf) of the interarrival time (in minutes) of the buses? (5%)

6. Let X and Y be joint Gaussian random variables with $\overline{X}=\overline{Y}=0$, $\sigma_X^2=9$, $\sigma_Y^2=4$, and the correlation coefficient $\rho_{X,Y}=0.4$, are transformed to new random variables V and W according to

V=X+Y

W=2X-Y

Find: (a) $\overline{W}$, (5%)

(b) covariance COV(X,V), (5%)

(c) σ_V^2, (5%)

(d) joint pdf of V and W. (5%)

7. Known that $t^2y''+ty'+t^2y=0$, $y(0)=1$, $y'(0)=0$,

Find: (a) the Laplace transform of y(t), (7%)

(b) the Laplace transform of $y(\lambda t)$. (3%)

8. Find: (a) the Laplace transform of $\left[e^{-2t}\int_0^t e^{3y}\sin 5y\,dy\right]$, (5%)

(b) the inverse Laplace transform of $\left[e^{-2s}\ell_n(1+\frac{1}{s^2})\right]$. (5%)