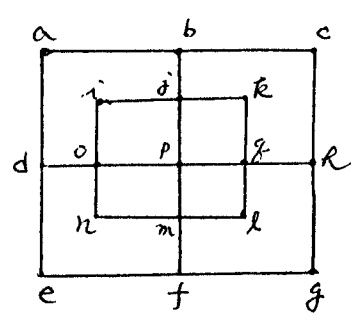


1. How many of the 9000 four-digit integers 1000, 1001, ..., 9998, 9999 have four digits that are either nondecreasing (as in 1347, 1236, and 7778) or nonincreasing (as in 6421, 6662, 7222)? (10%)
2. For the finite state machine given in the following table, determine its equivalent minimal machine using the equivalence-class partition method. (10%)

	next state		output	
	$x=0$	$x=1$	$x=0$	$x=1$
s_1	s_7	s_6	1	0
s_2	s_7	s_7	0	0
s_3	s_7	s_2	1	0
s_4	s_2	s_3	0	0
s_5	s_3	s_7	0	0
s_6	s_4	s_1	0	0
s_7	s_3	s_5	1	0
s_8	s_7	s_3	0	0

3. Prove that in a sequence of m integers there exists one or more consecutive terms with sum divisible by m . (10%)
4. Chengkung township sponsors four Boy Scout troops, each with 20 boys. If the head scoutmaster selects 50 of these boys to represent this township at the nation jamboree, what is the probability that his selection will include at least one boy from each of the four troops? (10%)
5. A young pair of rabbits (one of each sex) is placed on an island. A pair of rabbits does not breed until they are two months old. After they are two months old, each pair of rabbits produce another pair each month. Assume that no rabbits ever die. Find a recurrence equation for the number of pairs of rabbits on the island after n months and solve the equation. (10%)

6. Does the graph in the right have a Hamilton path? If it does, find such a path. If it does not, give a reason. (10%)



(背面仍有題目, 請繼續作答)

7. Television channels 2 through 13 are assigned to stations in Taiwan so that no two stations within 5 miles can operate on the same channel. Model this assignment of channels using some method based on graph theory. (10%)

8. A tree with n vertices is called graceful if its vertices can be labeled with the integers $1, 2, 3, \dots, n$ such that the absolute value of the difference of the labels of adjacent vertices are different. a) Show that the following tree is graceful. (5%)



b) prove that the n -vertex path, $n \geq 2$, is graceful (5%)

9. Let $R = M_2(\mathbb{Z})$ (the set of all 2×2 matrices with integer entries) and let S be the subset of R where $S = \left\{ \begin{bmatrix} x & x-y \\ x-y & y \end{bmatrix} \mid x, y \in \mathbb{Z} \right\}$
 prove that S is a subring of R . (10%)

10. In the following feedback network, we have a unit delay. This device causes the complement of the output f at time t to be fed back into the AND gate, along with new values for the input x, y , at the next unit of time, $t+1$. Complete the table. (10%)

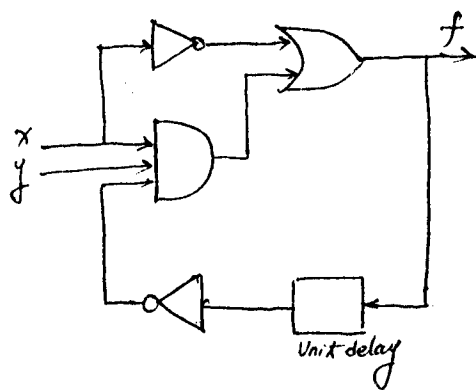


Table:

t	0	1	2	3	4	5	6
x	0	0	1	0	1	1	0
y	0	1	1	0	0	1	1
f	1	1	?	?	?	?	?