* **Useful constants** : $\varepsilon_0 = 10^{-9}/(36\pi)$ (F/m); $\mu_0 = 4\pi \times 10^{-7}$ (H/m); $\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 120\pi \approx 377\ (\Omega)$

1. **Maxwell's equations and wave equations:** (各3分)
   (a) Write the differential form of the Maxwell's equations.
   (b) Which term in the Maxwell's equations is the ***displacement current density*** term? Explain the physical meaning of the ***displacement current***.
   (c) The inhomogeneous wave equations of the E and H fields can be derived from (a) as follows.

$$\left(\nabla^2 - \mu\varepsilon\frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix}\bar{E}\\ \bar{H}\end{Bmatrix} = \begin{Bmatrix}\nabla\rho/\varepsilon + \mu\partial\bar{J}/\partial t\\ -\nabla\times\bar{J}\end{Bmatrix}$$

   Derive the time harmonic form of inhomogeneous wave equations (Helmholtz equation).
   (d) Write the time harmonic form of homogeneous wave equations (in source-free region).
   (e) Write the mathematical form of the E and H fields of an x-polarized uniform plane wave (TEM wave) propagating in the +z direction.
   (f) Prove that E-filed in (e) satisfy the equation: in (d).

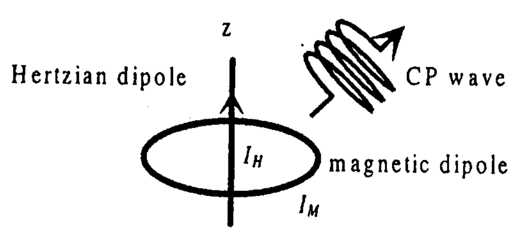
2. The far-zone E field of a Hertzian electric dipole of current I_H and length L along the z-axis is

$$E_\theta = j\eta_0\beta I_H L\left(\frac{e^{-j\beta r}}{4\pi r}\right)\sin\theta$$

and that of an element magnetic dipole of loop current I_M and radius b lying in the xy-plane is

$$E_\phi = \omega\mu_0\beta m\left(\frac{e^{-j\beta r}}{4\pi r}\right)\sin\theta \qquad m = I_M(\pi b^2) \qquad \text{(各5分)}$$

   (a) Determine the far-zone H fields of the Hertzian and the element magnetic dipoles.
   (b) Prove that a composite antenna consists of a Hertzian dipole and an element magnetic dipole can produce a circularly polarized wave . Also Indicate the necessary conditions.
   (c) Find the time-average radiation power density of a Hertzian dipole (L =1 cm, I_H =0.1 A) and an element magnetic dipole (b=0.5 cm, I_M =0.1A) at a distance r = 100 m and $\theta = 30^o$, at a frequency f =1 GHz. (Calculate the value in terms of unit mW/cm^2.)
   (d) Find the radiation power of a Hertzian dipole and an element magnetic dipole, respectively.
   (e) Find the radiation resistance of a Hertzian dipole and an element magnetic dipole, respectively.



Note : $\int_0^{2\pi} \sin^3\theta d\theta = 4/3$

(背面仍有題目,請繼續作答)

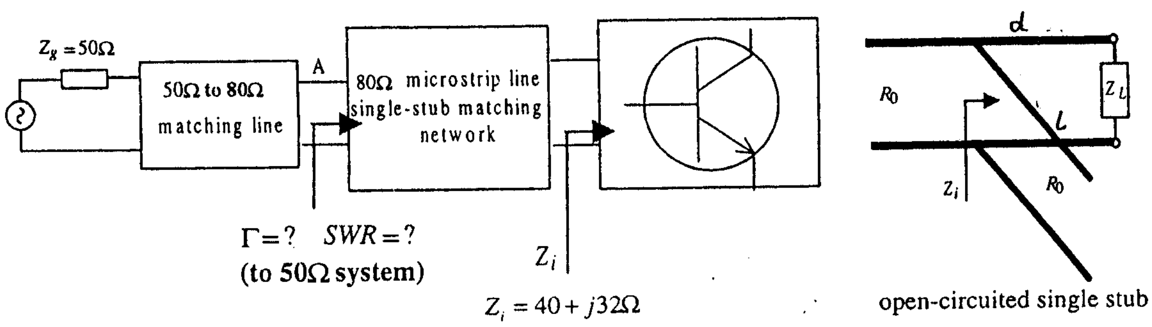
3. For the submarine communication under the seawater, extremely low frequency (ELF) must be used to increase the penetration of the EM waves into the conducting seawater. The dielectric constant and conductivity of the seawater are $\sigma = 4$ and $\varepsilon_r = 72$. Assuming a 45-Hz ELF TEM plane wave is incident from free space normally into the seawater
   (a) Determine the attenuation constant, the phase constant, the intrinsic impedance, the phase velocity, the wavelength, and the skin depth of the seawater. (6分)
   (b) Determine approximately the attenuation of the EM wave in the seawater per meter (in terms of dB/m). (3分)
   (c) Is the seawater a dispersive medium? Explain the physical meaning of dispersion. (5分)
   (d) If a plane wave is incident from seawater onto the water-air interface at 45°, will the wave be totally internally reflected? Prove your answer by assuming $\sigma \approx 0$ for the seawater to determine the critical angle first. (3分)

Note:

$$\gamma = \alpha + j\,\beta = j\,\omega\sqrt{\mu\,\varepsilon(1 - j\,\frac{\sigma}{\omega\varepsilon})} \;\Rightarrow\; \alpha = \omega\sqrt{\mu\,\varepsilon/2}\left[\sqrt{1+(\sigma/\omega\varepsilon)^2} - 1\right]^{1/2} \quad \beta = \omega\sqrt{\mu\,\varepsilon/2}\left[\sqrt{1+(\sigma/\omega\varepsilon)^2} + 1\right]^{1/2}$$

For good conductors $(\sigma/\omega\varepsilon) \gg 1 \Rightarrow \alpha \approx \sqrt{(\omega\mu\sigma)/2} \quad \beta \approx \sqrt{(\omega\mu\sigma)/2}$

4. Single-stub method is applied to microstrip line matching in RF/microwave circuit design. We want to design an input-matching network for a transistor amplifier shown in the following figure. Let the transistor input impedance be $(40 + j32\Omega)$ and it is treated as a one-port matching problem. The single-stub matching network uses the 80Ω microstrip line and an 80Ω-50Ω quarter-wavelength matching line is designed to the 50Ω system. (各4分)
   (a) Use the Smith Chart to find the length d of shorter length and the stub length l of the open-circuited stub in terms of wavelength.
   (b) If substrate dielectric constant ε_r =4.7 and the substrate thickness is d=1.2 mm, find the physical length and width of the microstrip lines (by using the parallel-plate formula to approximate the microstrip line) in (a).
   (c) Find the reflection coefficient Γ and SWR (to the 50Ω system) at the point A.
   (d) Find the physical length and width of the microstrip line of the 80Ω-50Ω quarter-wavelength matching line on the same substrate in (b).



5. **Waveguide Problem**

An automotive tunnel is rectangular in cross section (width a = 15m and height b = 5 m) and with metal walls. The tunnel is treated as a waveguide. (各4分)

(a) Find the lowest frequency of the radio wave that will propagate as a nonevanescent wave through this tunnel.

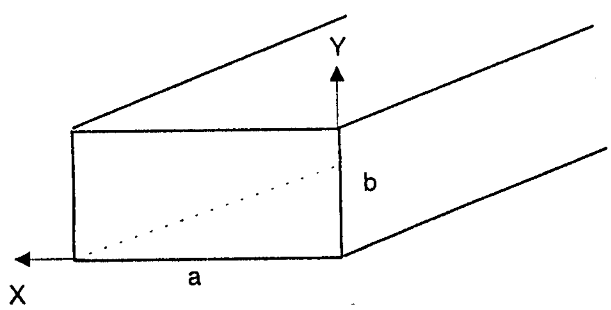
(b) Will the AM broadcast radio signal (550 - 1600 KHz) propagate in this tunnel? If yes, determine all the propagating modes of the lowest frequency AM signal.

(c) Will the 88-MHz FM broadcast radio signal propagate in this tunnel? If yes, determine the all propagating modes of the lowest frequency FM signal.

(d) Write the expression and plot the cross-sectional E and H field distribution of the dominant mode in the waveguide.

(e) Determine the wavelength λ_g and waveguide impedance Z_g of the signal with frequency 10% higher than the cutoff frequency of the dominant mode in this tunnel waveguide.

(f) Let the length of the tunnel be 40 m and one end is sealed with metallic wall (short-circuited), find the SWR of the dominant-mode wave with frequency 10% higher than the cutofffrequency at the beginning of the tunnel. (You can use the Smith chart.)



Waveguide TM mode fields	Waveguide TE mode fields
$E_z^0(x,y) = E_0 \sin(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$	$H_z^0(x,y) = H_0 \cos(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$
$E_x^0(x,y) = -\frac{\gamma}{h^2}(\frac{m\pi}{a})E_0 \cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$	$E_x^0(x,y) = \frac{j\omega\mu}{h^2}(\frac{n\pi}{b})H_0 \cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$
$E_y^0(x,y) = -\frac{\gamma}{h^2}(\frac{n\pi}{b})E_0 \sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$	$E_y^0(x,y) = -\frac{j\omega\mu}{h^2}(\frac{m\pi}{a})H_0 \sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$
$H_x^0(x,y) = \frac{j\omega\varepsilon}{h^2}(\frac{n\pi}{b})E_0 \sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$	$H_x^0(x,y) = \frac{\gamma}{h^2}(\frac{m\pi}{a})H_0 \sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$
$H_y^0(x,y) = -\frac{j\omega\varepsilon}{h^2}(\frac{m\pi}{a})E_0 \cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$	$H_y^0(x,y) = \frac{\gamma}{h^2}(\frac{n\pi}{b})H_0 \cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$

$$\gamma = j\beta = j\sqrt{\omega^2\mu\varepsilon - (\frac{m\pi}{a})^2 - (\frac{n\pi}{b})^2} \qquad h^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2$$

(背面仍有題目,請繼續作答)

NAME	TITLE	DWG. NO.
SMITH CHART FORM 82-BSPR(9-66)	KAY ELECTRIC COMPANY, PINE BROOK, N.J. ©1966. PRINTED IN U.S.A.	DATE

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