86 學年度 國立成功大學 碩士班招生考試	電機研究	所(甲、乙、丙、戊、己、庚) 工程數學	試題	共 1 頁 第 1 頁

1. Solve for the following differential equations.
   (a) $yy''+(y+1)(y')^2=0$. (5%)
   (b) $y''-4y=\sum_{n=1}^{\infty}\frac{1}{n}\sin(nx)$. (5%)

2. (a) Evaluate the integral $I=\int_0^{\infty}\sqrt{x}e^{-x^3}dx$. (5%)
   (b) Given that Laplace transform $L\{t^{-1/2}\}=(\pi/s)^{1/2}$, please find the value of $L\{t^{1/2}\}$. (5%)

3. For the differential equation $xy''-2xy'+2y=0$,
   (a) Find the general solution in power series form. (10%)
   (b) Check for the radius of convergence of the power series from (a). (5%)

4. Find the Fourier Transform of the following function
   (a) Unit step function $u(t)=\begin{cases}1 & t\ge 0\\ 0 & t<0\end{cases}$ (5%)
   (b) Delta pulse $d(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)$, where $\delta(t)$ is Dirac delta function. (5%)
   (c) $|F(\omega)|$ is the amplitude spectrum of $f(t)$. Determine and draw $|F(\omega)|$, where $f(t)=\frac{2\sin(at)}{t}$ (5%)
   (d) Let $g(t)=d(t)f(t)$ where $d(t)$ and $f(t)$ are defined in (b) and (c). In addition, let $0<T<\frac{\pi}{a}$. Determine and draw the amplitude spectrum of $g(t)$ (5%)

5. Solve the following Strum-Liouville problem on interval (0, R]
   $x^2y''+xy'+(\lambda x^2-n^2)y=0$;y(R)=0 where n is a nonnegative integer.
   (a) Find the eigenvalues and the corresponding eigenfucntions. Express eigenfunctions in terms of $J_\upsilon(x)$ and $Y_\upsilon(x)$. (10%)
   (b) In addition, we require that the solutions remain bounded as $x\to 0$ from the right. Determine the general solution of this problem. (5%)

6. Solve the following system of equations by using method of eigenvalues and eigenvectors, (10%)
   $y_1''=-5y_1+2y_2$;
   $y_2''=2y_1-2y_2$, where $y''=d^2y/dt^2$.

7. Expand each of the following functions in a Laurent series that converge for $0<|z|<R$ and determine the precise region of convergences:
   (a) $\frac{1}{z(1+z^2)}$,(5%) (b) $z\cos(\frac{1}{z})$. (5%)

8. Evaluate the following integral, where C is the ellipse $9x^2+y^2=9$ (counterclockwise) (10%):
$$\oint_C\left(\frac{ze^{\pi z}}{z^4-16}+ze^{\pi/z}\right)dz.$$

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