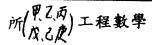
## 86 學年度 國立成功大學 碩士班招生考試

電機研究



試題 共 / 頁

- 1. Solve for the following differential equations.
  - (a)  $yy''+(y+1)(y')^2=0$ . (5%)
  - (b)  $y''-4y = \sum_{n=1}^{\infty} \frac{1}{n} \sin(nx)$ . (5%)
- 2. (a) Evaluate the integral  $I = \int_{0}^{\infty} \sqrt{x}e^{-x^3} dx$ . (5%)
  - (b) Given that Laplace transform  $L\{t^{-1/2}\}=(\pi/s)^{1/2}$ , please find the value of  $L\{t^{1/2}\}$ . (5%)
- 3. For the differential equation xy''-2xy'+2y=0,
  - (a) Find the general solution in power series form. (10%)
  - (b) Check for the radius of convergence of the power series from (a). (5%)
- 4. Find the Fourier Transform of the following function
  - (a) Unit step function  $u(t) = \begin{cases} 1 & t \ge 0 \\ 0 & t < 0 \end{cases}$  (5%)
  - (b) Delta pulse  $d(t) = \sum_{n=-\infty}^{\infty} \delta(t nT)$ , where  $\delta(t)$  is Dirac delta function. (5%)
  - (c)  $|F(\omega)|$  is the amplitude spectrum of f(t). Determine and draw  $|F(\omega)|$ , where  $f(t) = \frac{2\sin(at)}{t}$  (5%)
  - (d) Let g(t) = d(t)f(t) where d(t) and f(t) are defined in (b) and (c). In addition, let  $0 < T < \frac{\pi}{a}$ . Determine and draw the amplitude spectrum of g(t) (5%)
- 5. Solve the following Strum-Liouville problem on interval (0, R]  $x^2y'' + xy' + (\lambda x^2 n^2)y = 0; y(R) = 0 \text{ where n is a nonnegative integer.}$ 
  - (a) Find the eigenvalues and the corresponding eigenfunctions. Express eigenfunctions in terms of  $J_{\nu}(x)$  and  $Y_{\nu}(x)$ . (10%)
  - (b) In addition, we require that the solutions remain bounded as  $x \rightarrow 0$  from the right. Determine the general solution of this problem. (5%)
- 6. Solve the following system of equations by using method of eigenvalues and eigenvectors, (10%)

$$y_1'' = -5y_1 + 2y_2;$$
  
 $y_2'' = 2y_1 - 2y_2,$  where  $y'' = d^2y/dt^2.$ 

7. Expand each of the following functions in a Laurent series that converge for 0 < |z| < R and determine the precise region of convergences:

(a) 
$$\frac{1}{z(1+z^2)}$$
, (5%) (b)  $z\cos(\frac{1}{z})$ . (5%)

8. Evaluate the following integral, where C is the ellipse  $9x^2 + y^2 = 9$  (counterclockwise) (10%):

$$\oint \left(\frac{ze^{\pi z}}{z^4-16}+ze^{\pi/z}\right)dz.$$