1. Consider the vector space P_2 of polynomials of degree at most 2. Let $T : P_2 \rightarrow P_2$ be a linear transformation such that $T(1) = 3 + 2x + x^2$, $T(x) = 2$, $T(x^2) = 2x^2$. Let $T^i(a) = T(T^{i-1}(a))$, for $a \in P_2$ and $i \geq 2$, find $T^k(\ x+2\ )$ in terms of k. (15%)

2. Let $v_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}$, $v_2 = \begin{bmatrix}2\\1\\1\end{bmatrix}$, $v_3 = \begin{bmatrix}1\\0\\1\end{bmatrix}$, $v_4 = \begin{bmatrix}3\\0\\-1\end{bmatrix}$ be vectors in R^3. Let $W_1 = SP\,(v_1,\ v_2) = \{\ r_1v_1 + r_2v_2 \mid r_1,\ r_2 \in R\ \}$ and $W_2 = SP\,(v_3,\ v_4)$. Find a set of generating vectors for $W_1 \cap W_2$. (10%)

3. Let A be an n×n matrix and I be the n×n identity matrix.
(a) Prove that AA^T and A^TA have the same eigenvalues. (5%)
(b) If λ is an eigenvalue of A with a corresponding eigenvector v. Find the eigenvalue and corresponding eigenvector of A+rI for a scalar r in terms of λ and v. (10%)

4. A current having a Rayleigh probability density function

$$f_I(i) = \begin{cases} (i/a^2)\exp(-i^2/2a^2), & i > 0 \\ 0, & \text{otherwise.} \end{cases}$$

is passed through a resistor having a resistance of 2π ohms. If the mean value of the current is $E(I)=a(\pi/2)^{1/2} = 2$ amperes, and the mean squared current is $E(I^2)=2a^2$, what is the mean value of the power dissipated in the resistor? (15%)

5. The random process X(t) has the autocorrelation function $R_X(t)=15\ e^{-2|t|}$. The random process Y(t) is Y(t)=X(t)-3.

(a) What is the mean of Y(t)? (5%)

(b) What is the autocorrelation function for Y(t)? (5%)

(c) What is the variance of Y(t)? (5%)

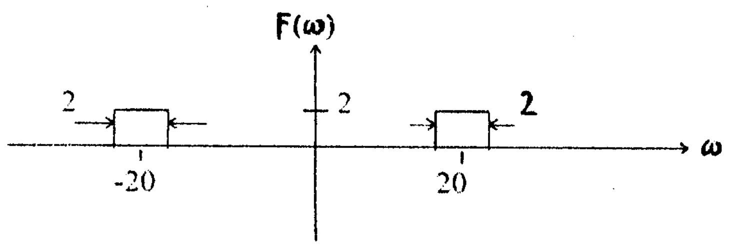
(d) What is the crosscorrelation $R_{XY}(t)$? (5%)

6. A certain function of time, f(t), has a Fourier transform as shown.

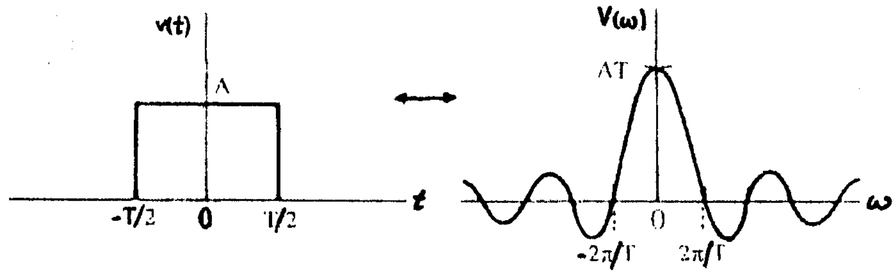
(a) Sketch the Fourier transform of f(3t). (3%)

(b) Sketch the Fourier transform of $[f(t)]^2$. (3%)

(c) Sketch the Fourier transform of [f(t)cos(15t)]. (4%)



7. It is known that v(t)=A.rect(t/T) and V(ω)=AT.sinc(ωT) are Fourier Transform pairs, as illustrated.



(a) By modulation property $v(t)\cos(\omega_o t) \leftrightarrow (1/2)[V(\omega-\omega_o) + V(\omega+\omega_o)]$, sketch the Fourier spectrum of $v(t)\cos(\omega_o t)$. (5%)

(b) Sketch the waveform $v(t)\cos(\omega_o t) * v(t)\cos(\omega_o t)$; that is, the convolution of $v(t)\cos(\omega_o t)$ with itself. (10%)