(20%) 1. A white Gaussion signal with two-sided PSD = $10^{-6}\,\mathrm{W/Hz}$ is passed through an ideal bandpass filter with a bandwidth of 100KHz centered at 100 MHz.

(a) What is the power of the output signal x(t) from this BPF? (5%)

(b) We may represent x(t) by

$x(t) = x_c(t)\cos(2\pi f_0 t + \theta) - x_s(t)\sin(2\pi f_0 t + \theta)$, where $f_0 = 100\mathrm{MHz}$.

What are the powers of $x_c(t)$ and $x_s(t)$? (5%)

(c) What is the autocorrelation function of $x_c(t)$? (5%)

(d) What is the joint probability density function of $x_s(t)$ at t = 10 sec and t = 11 sec? (5%)

(10%) 2. The frequency range of commercial AM broadcast is 540-1600 KHz. The local oscillator frequency f_{LO} of a superheterodyne AM receiver is chosen such that $f_{LO} > f_{RF}$ (the radio frequency). It is required that image frequency be outside of the broadcast frequency region. Determine the minimum required f_{IF} (the intermediate frequency) and the range of the corresponding f_{LO}.

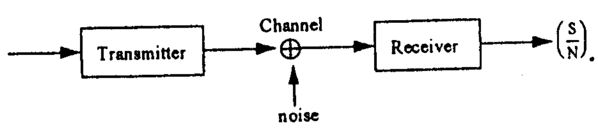
(20%) 3. The normalized message signal $m_n(t)$ has a bandwidth of 5KHz and power of 0.2W. The output power of the transmitter is 20 KW. The channel has a bandwidth of 200 KHz and attenuation of 93dB. The noise is white with two-sided PSD = $10^{-14}\,\mathrm{W/Hz}$.

(a) IF the transmitter is an AM modulator with modulation index of 0.5, what is the highest possible $(\frac{S}{N})_o$? (7%)

(b) IF the transmitter is an SSB modulator, what is the highest possible $(\frac{S}{N})_o$? (5%)

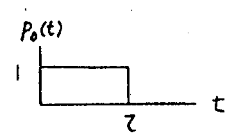
(c) IF the transmitter is an FM modulator, what is the highest possible $(\frac{S}{N})_o$? (8%)

(Hint : Assuame $(SNR)_{FM} = 3D^2\overline{m_n^2}\frac{P_T}{N_0 W}$, where W is the bandwidth of the message signal and D is the deviation ratio).



(16%) 4. A signal s(t) corrupted by an additive white noise n(t) is to be detected by the following receiver.

Assume s(t) = $p_0(t)$, where $p_n(t) = rect(\frac{t-\frac{1}{2}\tau-nD}{\tau})$.



The noise power spectral density $G_n(f)$ is equal to $\frac{N_0}{2}$, and h(t) is a causal filter.

(a) What is the maximum signal-to-noise ratio (SNR) that can be obtained by the receiver. Assume T is large enough. (2%)

(b) What is the minimum T, denoted by T_{min}, required in (a). (2%)

(c) What is the optimal filter $h_{opt}(t)$ that produces the maximum SNR at T_{min}. (2%)

(d) Plot the output response $s_0(t) = s(t) * h_{opt}(t)$. (2%)

(e) if $T = \frac{\tau}{2}$, what is the maximum SNR that can be obtained by using a causal filter. (2%)

(f) What is the corresponding optimal filter in (e). (2%)

(g) Repeat (d) for s(t) = $p_0(t) + p_1(t) + p_2(t)$. (2%)

(h) Under what condition there is no intersymbol interference in (g). (2%)

(20%) 5. (a) Give the signal constellation of the 16-QAM (or QASK). Show the block diagram of a correlation detector for 16-QAM. (4%)

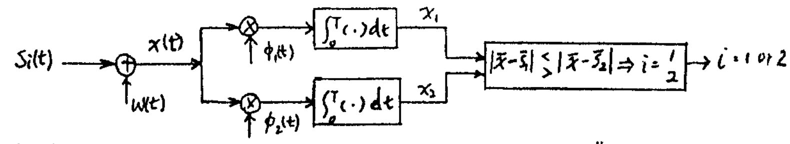
(b) If the encoded DPSK sequence received is 1100101011, what is the corresponding messages recovered? In your answer, you have to explain how you obtain your answer. (4%)

(c) A digital transmission system is designed to transmit data at R_b bits per second. What is the minimum bandwidth required? How you do it? (4%)

(d) Plot the power spectral density of M-ary PSK for M=2,4,8. Estimate the modulation speed for each. (4%)

(e) List the advantages and disadvantages of the signaling scheme with correlative coding over the signaling scheme without. Give the rationale for each item you list. (4%)

(14%) 6. A correlator receiver used to determine $s_1(t)$ or $s_2(t)$ was sent is shown below.



w(t) is the white Gaussian noise with power spectral density $\frac{N_0}{2}$. The energies of $s_1(t)$ and $s_2(t)$ are the same and equal to E. $\phi_1(t)$ and $\phi_2(t)$ form an orthonormal basis in $0 \le t \le T$ and

$$s_1(t) = s_{11}\phi_1(t) + s_{12}\phi_2(t),\ 0 \le t \le T$$

$$s_2(t) = s_{21}\phi_1(t) + s_{22}\phi_2(t),\ 0 \le t \le T$$

$$< s_i(t), \phi_j(t) > = \int_0^T s_i(t)\phi_j(t)dt$$

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},\ \bar{s}_1 = \begin{bmatrix} s_{11} \\ s_{12} \end{bmatrix},\ \bar{s}_2 = \begin{bmatrix} s_{21} \\ s_{22} \end{bmatrix}$$

(a) Calculate the variance of random variable x_1 given that $s_1(t)$ is sent. (5%)

(b) Show that the minimum error probability of the system, p_e, is given by (5%)

$$p_e = \frac{1}{2} erfc\{\alpha[SNR(1-\rho_{12})]^{\beta}\}$$

Where α and β are two constants, $SNR = \frac{E}{N_0}$, ρ_{12} is the correlation coefficient between $s_1(t)$ and $s_2(t)$ given by $\rho_{12} = \frac{1}{E}\int_0^T s_1(t)s_2(t)dt$

and erfc(u) is given by $erfc(u) = \frac{2}{\sqrt{\pi}}\int_u^{\infty} e^{-z^2}dz$

(c) Find α, β and ρ_{12} for BPSK. (4%)