1. Suppose that a differential equation was found to have the solutions 8sinh(3x) and 12cosh(3x). Please determine the differential equation fits the requirement. (5%)

2. Given that $y_1 = \cos(\ln x)$ and $y_2 = \sin(\ln x)$ are linearly independent solutions of the homogeneous differential equation $x^2y''+xy'+y=0$ on $(0,\infty)$. Find the general solution of the equation $x^2y''+xy'+y=\sec(\ln x)$ and state an interval of validity. (10%)

3. For the nonlinear equation $y'+ay^2+by+c=0$ with arbitrary constants a, b, and c,
   (a) Try to apply substitution to change the equation into a constant coefficient linear second order differential equation. (5%)
   (b) Given that $a=1$, $b=2$, $c=1$, and $y(1)=0$, solve for the equation. (5%)

4. (a) Apply the convolution theorem to prove that $\mathcal{L}\left\{\int_0^t f(x)dx\right\} = F(s)/s$, where $F(s)=\mathcal{L}\{f(t)\}$. (5%)
   (b) Evaluate for the transform $\mathcal{L}\left\{\int_0^{t-a} f(x)dx\right\}$. (5%)

5. Given the function shown as follows:

$$f(x)=\begin{cases} 1+x, & -1<x\le 0 \\ -(x-1), & 0<x\le 1 \\ 0, & otherwise \end{cases}$$

   (a) Calculate the Fourier integral representation of the above function. (5%)
   (b) Compute the value of $\int_0^\infty \frac{\cos\frac{\omega}{2}-\cos\frac{\omega}{2}\cos\omega}{\omega^2}d\omega$. (10%)

6. If the general solution to the equation of $x^2y''+xy'+(x^2-\nu^2)y=0$ is given as $y(x)=aJ_\nu(x)+bY_\nu(x)$, please find the general solution in terms of $J_\nu(x)$ and $Y_\nu(x)$ to the following equation, $y''=(2x^{-2}-x^{-1})y+x^{-1}y'$. (10%)

7. For the differential equation listed below:
   $y=xy'+x^2y'-x^2y''$
   determine the general solution in power series form. (10%)

8. Determine the following statements as being true or false. Please put "T" (to denote the true statement) and "F" (to present the false the false statement) on your answer sheet. *No* proof or explanation is needed. The grade for each statement is three points. If your answer is incorrect, extra three points will be subtracted from this question. (0% ~ 30%)

(背面仍有題目,請繼續作答)

(1) Let A and B be $n \times n$ matrices, $x_1 \neq 0$ and $x_1 \in R^n$. Then $Ax_1 = Bx_1$ if and only if $A = B$.

(2) If A is nonsingular, then $\det(A^{-1})A = \operatorname{adj} A^{-1}$

(3) Let R^+ present the set of positive real numbers. Define the operation of scalar multiplication, denoted $\otimes$, by $\alpha \otimes x = x^\alpha$ (for example, $3 \otimes 5 = 5^3$) for each $x \in R^+$ and for any real number α. Define the operation of addition, denoted $\oplus$, by $x \oplus y = xy$ (for instance, $3 \oplus 5 = 15$) for all $x, y \in R^+$. Then R^+ is a vector space.

(4) The transition matrix representing the change of coordinates from $[1,\ e^x,\ e^{-x}]$ to $[1, \cosh x, \sinh x]$ is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & -0.5 \end{bmatrix}$.

(5) There do not exist $n \times n$ matrices A and B such that $AB - BA = I$.

(6) Let A be a $n \times n$ matrix and let λ be an eigenvalue of multiplicity $n-1$. If $A - \lambda I$ has rank 1, then A is defective.

(7) Let $f(z) = u(x,y) + i\,v(x,y)$ be differentiable on an open disk D and assume that u and v are continuous first partial derivatives on D. Then $f(z)$ is constant on D if and only if $|f(z)|$ is constent on D.

(8) The following real integrals are all correct: $\int_{-\infty}^{\infty} \frac{\cos(2x)}{(x^2+9)(x^2+25)}dx = 0$, $\int_0^{\infty} \frac{x^{1/3}}{x(x^2+1)}dx = \pi$, and $\int_0^{\infty} e^{-x^2}\cos(2x)dx = \frac{\sqrt{\pi}}{2}e^{-1}$.

(9) If $f(z)$ is differentiable at z_0 and $g(z)$ has a pole of order m at z_0, then $f(z)g(z)$ gas a pole of order $m+1$ at z_0.

(10) A linear fractional transformation is a function given by $T(z) = \frac{az+b}{cz+d}$ in which a, b, c and d are complex numbers and $ad-bc \neq 0$. Then T maps a circle to a circle, and a line to a line.