1. A firm produces ten thousand ASIC chips. A sample of 100 is taken and 5 are found to be defective. What is the probability that this occurs if there are k defective chips. (10%)

2. (a) Let $A = \{x, a, b, c, d\}$. How many closed binary operations f on A satisfy $f(a,b)=c$ and have x as an identity? (5%)

   (b) In the following Pascal program segment, the integer variables i, j, n and sum are declared earlier in the program. Define the time-complexity function as the number of times the statement sum := sum + 1 is executed. Determine the best "big-Oh" form for f. (5%)

```
Begin
  sum := 0;
  For i := 1 to n do
    Begin
      j := n;
      while j > 0 do
        Begin
          sum := sum + 1;
          j := j div 2;
        end
    end
end;
```

3. Construct a state diagram for a finite state machine with input and output alphabets $= \{0, 1\}$ that recognizes all string in the language $\{0,1\}^*\{00\} \cup \{0,1\}^*\{11\}$. (10%)

4. A relation R on a set A is called irreflexive if for all $a \in A$, $(a,a) \notin R$.

   (a) Let $R \neq \phi$. Prove that if R satisfies any two of the following properties: irreflexive, symmetric, and transitive, then it can not satisfy the third.

   (b) If $|A| = n \geq 1$, how many different relations on A are irreflexive? (10%)

5. Find the number of positive integers n where $1 \leq n \leq 3000$ and n is not a perfect square, cube, or fifth power. (10%)

6. (a) Find the generating function of sequence 1, 2, 4, 8, 16, ... (7%)
(b) Use the generating function approach to obtain a formula for $\sum_{i=0}^{n} i(i-1)$. (7%)

7. Find a recurrence equation for the number of regions into which the plane is divided by n straight lines if every pair of lines intersect, but no three lines meet at a common point. (8%)

8. If $G=(V,E)$ is a directed graph with $|V|=v$, $|E|=e$, and no loops, prove $e \le v^2 - v$ and state the corresponding inequality for the case when G is undirected. (8%)

9. Let $(Q, \oplus, \odot)$ denote the field where $\oplus$ and $\odot$ are defined by

$$a \oplus b = a + b - k, \quad a \odot b = a + b + (ab/m)$$

for fixed elements k, m $(\neq 0)$ of Q. Find the relation between k and m. (10%)

10. In how many ways can you paint the eight regions of the square shown in the following if five colors are available? (10%)