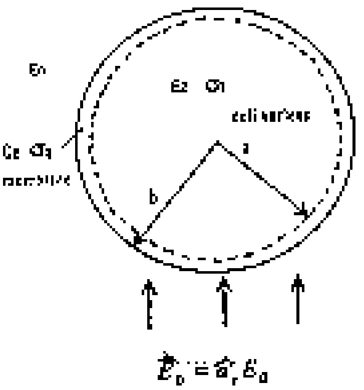
1. **Maxwell's Equations and Wave Equations: (25%)**
   (a) Write the Maxwell's equations in differential form. (3%)
   (b) Write the equation of continuity. (3%)
   (c) Which term in Maxwell's equations is the ***displacement current density*** ? How did Maxwell add to this term to make the equations consistent with the principle of charge conservation (equation of continuity)? Briefly explain the physical meaning of the displacement current. (5%)
   (d) Derive the inhomogeneous wave equations for the **E** and **H** fields from Maxwell's equations. (4%)
   (e) Derive the time-harmonic (source-free) Helmholtz's equation for the **E** and **H** fields. (4%)
   (f) Write the **E** and **H** fields of an x-polarized plane wave propagating in the +z direction. (3%)
   (g) Prove that **E**-field in (f) satisfies the equation in (e). (3%)

2. A biological cell (生物細胞) modeled as a simple two-layer lossy sphere with $(\varepsilon_1, \sigma_1)$ in the cell membrane (細胞膜) and $(\varepsilon_2, \sigma_2)$ in the cell nucleus (細胞核) is placed in an initially uniform E-filed $\vec{E}_0 = \hat{a}_z E_0$. The induced field inside the cell can be obtained as

$$\begin{cases} \vec{E}_1(R,\theta) = \hat{a}_R \cos\theta(E_0 - A_1 + 2B_1/R^3) + \hat{a}_\theta \sin\theta(-E_0 + A_1 + B_1/R^3) \\ \qquad a < R < b \\ \vec{E}_2(R,\theta) = \hat{a}_z(E_0 - A_2) = \hat{a}_R \cos\theta(E_0 - A_2) + \hat{a}_\theta \sin\theta(-E_0 + A_2) \\ \qquad 0 < R < a \end{cases}$$

(where (A_1, B_1, A_2) are functions of $(\varepsilon_1, \sigma_1, \varepsilon_2, \sigma_2, \varepsilon_0)$)

A simple model of a biological cell



   (a) From the current-density boundary condition, find the two equations of the relation between (A_1, B_1, A_2). (5%)
   (b) Determine the surface charge distribution $\rho_s(a,\theta)$ at the boundary of the cell membrane and the cell nucleus. (5%)

3. A uniform plane wave from a dielectric medium ($\varepsilon_r = 4$) is incident <u>obliquely</u> on the boundary of the dielectric and air. Let the average power density p_i of the incident plane wave be 1 W/m^2.
   (a) Determine the critical angle θ_c of the total reflection. (5%)
   (b) Determine the average power densities of the <u>transmitted wave</u> at an incident angle 60°. (5%)

4. A microstrip line has a characteristic impedance of 50Ω and a substrate with dielectric constant $\varepsilon_r = 9$ and thickness $d = 1$ mm. Use the simple <u>parallel-plate formula</u> to determine: (25%)
   (a) the required width w of the microstrip line. (5%)
   (b) the physical length of a $\lambda/2$ microstrip line at 10 GHz. (5%)
   (c) the input impedance, reflection coefficient and VSWR of a microstrip line with length 0.25 mm and load impedance $25\Omega + j25\Omega$ at 10 GHz (by using the Smith chart). (10%)
   (d) If the microstrip line ($\varepsilon_r = 9$ & $d = 1$ mm) is used to design a matched section for a 50-Ω system to a load impedance 25Ω at 10 GHz, determine the the required width w and length. (5%)

5. A communication satellite is in a synchronous orbit 36,000 km above the earth. The satellite transmitter power is 100 W and the satellite antenna with gain 0 dB and radiation efficiency 0.6 at 10 GHz radiates a <u>circularly polarized (CP) wave</u>. The earth station receiving antenna with gain 40 dB and radiation efficiency 0.6 is a <u>linearly polarized antenna</u>. The receiving antenna is connected to the receiver through a rectangular waveguide with cross-section $a \times b$, $a = 2\ cm, b = 1\ cm$. (30%)

(a) Determine the time-average power density of the wave radiated from the satellite antenna at the earth station.

(b) Determine the received power of the earth station antenna (assuming matched load and polarization mismatch loss 3dB) by using the free-space power transmission formula,

$\frac{P_L}{P_t} = \frac{G_t \eta_t G_r \eta_r \lambda^2}{(4\pi R)^2} p$ P_t = transmitter power, P_L = received power to the matched load,

$\eta_t\ (\eta_r)$ = transmitting (receiving) antenna radiation efficiency, p = polarization mismatch loss

$G_t\ (G_r)$ = transmitting (receiving) antenna gain

(c) Let the ϕ component of the far-zone E field of the satellite-antenna radiated wave be

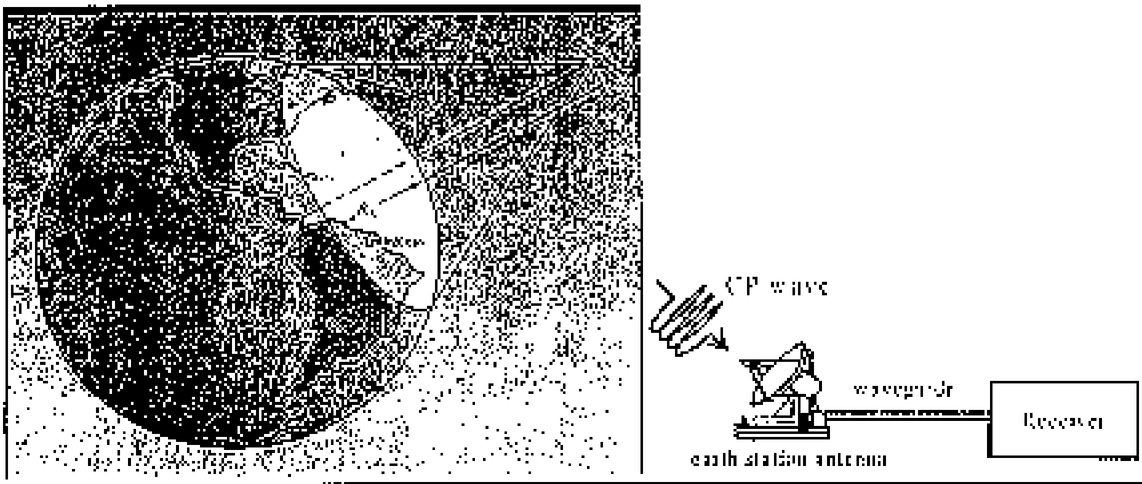
$E_\phi = E_0 \left(e^{-jkr}/4\pi R\right) f(\theta,\phi)$ ($f(\theta,\phi)$ is the antenna pattern function)

Write all other components of the E and H fields (either right-hand or left-hand CP)

(d) Determine the cutoff frequency of the dominant mode of the receiver waveguide and all modes that will propagate along the waveguide at the operating frequency (10 GHz).

(e) Determine the guided wavelength and the wave impedance of the waveguide dominant mode at the operating frequency (10 GHz).

(f) Write the E and H field distribution of the dominant mode and plot the cross-sectional field distribution.



TM mode

$$\begin{cases} E_z^0(x,y) = E_0 \sin(m\pi x/a)\sin(n\pi y/b) \\ E_x^0(x,y) = -\frac{\gamma}{h^2}(m\pi/a)E_0\cos(m\pi x/a)\sin(n\pi y/b) \\ E_y^0(x,y) = -\frac{\gamma}{h^2}(n\pi/b)E_0\sin(m\pi x/a)\cos(n\pi y/b) \\ H_x^0(x,y) = \frac{j\omega\varepsilon}{h^2}(n\pi/b)E_0\sin(m\pi x/a)\cos(n\pi y/b) \\ H_y^0(x,y) = -\frac{j\omega\varepsilon}{h^2}(m\pi/a)E_0\cos(m\pi x/a)\sin(n\pi y/b) \end{cases}$$

TE mode

$$\begin{cases} H_z^0(x,y) = H_0 \cos(m\pi x/a)\cos(n\pi y/b) \\ E_x^0(x,y) = \frac{j\omega\mu}{h^2}(n\pi/b)H_0\cos(m\pi x/a)\sin(n\pi y/b) \\ E_y^0(x,y) = -\frac{j\omega\mu}{h^2}(m\pi/a)H_0\sin(m\pi x/a)\cos(n\pi y/b) \\ H_x^0(x,y) = \frac{\gamma}{h^2}(m\pi/a)H_0\sin(m\pi x/a)\cos(n\pi y/b) \\ H_y^0(x,y) = \frac{\gamma}{h^2}(n\pi/b)H_0\cos(m\pi x/a)\sin(n\pi y/b) \end{cases}$$

$$\gamma = j\beta = j\sqrt{\omega^2\mu\varepsilon - (m\pi/a)^2 - (n\pi/b)^2} \quad h^2 = (m\pi/a)^2 + (n\pi/b)^2$$

$$\lambda_g = \lambda\Big/\sqrt{1-(f_c/f)^2} \quad Z_{TM} = \eta\sqrt{1-(f_c/f)^2} \quad Z_{TE} = \eta\Big/\sqrt{1-(f_c/f)^2}$$

$\varepsilon_0 = 10^{-9}/(36\pi)$ (F/m); $\mu_0 = 4\pi \times 10^{-7}$ (H/m); $\sqrt{\mu_0/\varepsilon_0} = 120\pi$ (Ω)

Some Useful Vector Identities

$$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A} = \mathbf{C}\cdot\mathbf{A}\times\mathbf{B}$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$$

$$\nabla(\psi V) = \psi\nabla V + V\nabla\psi$$

$$\nabla\cdot(\psi\mathbf{A}) = \psi\nabla\cdot\mathbf{A} + \mathbf{A}\cdot\nabla\psi$$

$$\nabla\times(\psi\mathbf{A}) = \psi\nabla\times\mathbf{A} + \nabla\psi\times\mathbf{A}$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$$

$$\nabla\cdot\nabla V = \nabla^2 V$$

$$\nabla\times\nabla\times\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$

$$\nabla\times\nabla V = 0$$

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0$$

$$\int_V \nabla\cdot\mathbf{A}\,dv = \oint_S \mathbf{A}\cdot d\mathbf{s} \quad \text{(Divergence theorem)}$$

$$\int_S \nabla\times\mathbf{A}\cdot d\mathbf{s} = \oint_C \mathbf{A}\cdot d\boldsymbol{\ell} \quad \text{(Stokes's theorem)}$$

Gradient, Divergence, Curl, and Laplacian Operations

Cartesian Coordinates (x, y, z)

$$\nabla V = \mathbf{a}_x\frac{\partial V}{\partial x} + \mathbf{a}_y\frac{\partial V}{\partial y} + \mathbf{a}_z\frac{\partial V}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \mathbf{a}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{a}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{a}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical Coordinates (r, ϕ, z)

$$\nabla V = \mathbf{a}_r\frac{\partial V}{\partial r} + \mathbf{a}_\phi\frac{\partial V}{r\,\partial\phi} + \mathbf{a}_z\frac{\partial V}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{\partial A_\phi}{r\,\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \frac{1}{r}\begin{vmatrix} \mathbf{a}_r & \mathbf{a}_\phi r & \mathbf{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \mathbf{a}_r\left(\frac{\partial A_z}{r\,\partial\phi} - \frac{\partial A_\phi}{\partial z}\right) + \mathbf{a}_\phi\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \mathbf{a}_z\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates (R, θ, ϕ)

$$\nabla V = \mathbf{a}_R\frac{\partial V}{\partial R} + \mathbf{a}_\theta\frac{\partial V}{R\,\partial\theta} + \mathbf{a}_\phi\frac{1}{R\sin\theta}\frac{\partial V}{\partial\phi}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \mathbf{a}_R & \mathbf{a}_\theta R & \mathbf{a}_\phi R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix} = \mathbf{a}_R\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]$$
$$+ \mathbf{a}_\theta\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial\phi} - \frac{\partial}{\partial R}(RA_\phi)\right]$$
$$+ \mathbf{a}_\phi\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial\theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

(背面仍有題目,請繼續作答)

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