1. Let A be an $n \times n$ matrix which contains submatrices R and S such that $A = \left[\begin{array}{c|c} R & O \\ \hline O & S \end{array}\right]$, where R is $r \times r$ matrix and S is $s \times s$ matrix. Prove that det (A) = det (R) * det (S). (10%)

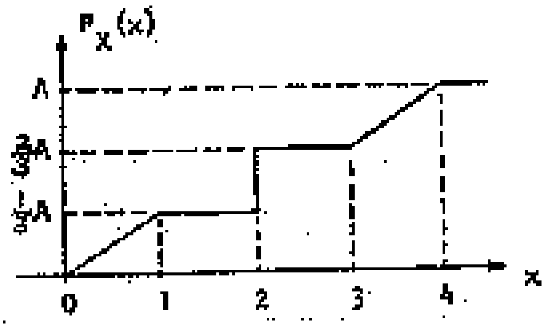
2. Explain the following terminologies in Linear Algebra.
   (a). Isomorphism; (3%)
   (b). Indempotent matrix; (2%)
   (c). Gram-Schmidt Process. (5%)

3. A Fibonacci sequence $[F_0, F_1, F_2, \ldots]$ is defined by $F_0 = 0$, $F_1 = 1$, and $F_i = F_{i-1} + F_{i-2}$ for all $i \geq 2$. Find F_k in terms of k for large value of k. (10%)

4. Find the discrete Fourier Transformation of the sequence {1, 1, 0, 0}. (10%)

5. Determine the Fourier transformation of $x(t) = \exp(-\alpha|t|)$, where $\alpha > 0$. (10%)

6. A random variable X has a cumulative distribution function as shown below: (20%)



(a). What is the value of A ?
(b). Sketch the probability density function.
(c). Find the expectation value E[X].
(d). What is the probability that X < 2 ?

7. A digital data transmission system selects one of the digits $X_i = 0, 1, 2,$ or 3 to transmit through a channel with prior probabilities 0.4, 0.3, 0.2, and 0.1, respectively. The conditional probabilities of receiving the digits $Y_j = 0, 1, 2,$ or 3 given that a $X_i = 0, 1, 2,$ or 3 is transmitted are given in the following table. (15%)

X_i	Y_j = 0	1	2	3
0	0.95	0.02	0.02	0.01
1	0.005	0.98	0.005	0.01
2	0.01	0.01	0.97	0.01
3	0.02	0.03	0.02	0.93

Obtain the following posterior probabilities:
(a). $P(X_i=2 \mid Y_j=2)$; (b). $P(X_i=1 \mid Y_j=2)$; and (c). $P(X_i=3 \mid Y_j=1)$.

8. Probability density functions for the two independent random variables X and Y are

$$f_X(x) = ae^{-ax}\,u(x)$$
$$f_Y(y) = (a^3/2)y^2e^{-ay}\,u(y),$$

where a=3. If Z = X + Y, what is $f_Z(z)$? (15%)