1. (a) Solve $(x^2+4)y' = 2x - 8xy$ subject to $y(0)=1$. (5%)
   (b) Solve the differential equation $y''+4y=\cos^2 x$. (5%)
   (c) Find a power series representation of $\cosh x$. (5%)

2. An exact differential equation $A(x)y''+B(x)y'+C(x)y=0$ can be expressed in the form $[A(x)y']+[F(x)y]'=0$ for some $F(x)$.
   (a) Show that if the equation is exact, then $A''(x)-B'(x)+C(x)=0$. (5%)
   (b) Solve for the general solution of the equation. (5%)

3. Use the Laplace transform to solve the problem. (10%)

$$y''+2ty'-4y=1;\ y(0)=y'(0)=0$$

4. Evaluate
   (a) $\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta)^2}$ $(a>b>0)$. (10%)
   (b) $\int_0^{\infty} \frac{\ln x}{\sqrt{x}(a^2+x^2)}dx$. (10%)

5. (a) Use Rodrigues' formula $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$ to find the expansion of the function

$$f(x)=\begin{cases}1 & \text{if } 0\le x\\ 0 & \text{if } x<0\end{cases}$$

   in terms of Legendre polynomials. (10%)
   (b) Expand the above function, $f(x)$, in a Fourier series in the interval [-1, 1]. (5%)I

6. Let $A=\begin{bmatrix}1 & 0 & 3\\ 0 & 1 & 1\\ 1 & 5 & 8\end{bmatrix}$. Please find the bases for $N(A)$ (the null space of A), $R(A)$ (the range space of A), $N(A^T)$, $R(A^T)$, $N(A^T)^{\perp}$ (the orthogonal complement of $N(A^T)$), and $R(A^T)^{\perp}$. Please also indicate which spaces are equal. (15%)

7. Please show that an $n\times n$ matrix A is diagonalizable if and only if A has n linearly independent eigenvectors. (15%)

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