1. X is a Gaussian random variable with a mean of 300 and a standard deviation of 50. Two events are defined as follows:

$$A = (200 < X < 325), \quad \text{and} \quad B = (280 < X < 400).$$

Use this information to calculate $P(A|B)$, and compare that with $P(A)$.

(10%)

2. Messages arrive at a computer at an average rate of 15 messages per second. The number of messages that arrive in 1 second is known to be a Poisson random variable.

a). Find the probability that no messages arrive in 1 second.　(7%)

b). Find the probability that more than 10 messages arrive in a 1-second period.　(8%)

3. A random variable X is Gaussian with zero mean and a variance of 1.0. Another random variable Y is obtained from $Y = X^3$.

(a). Write the probability density function for the random variable Y;　(5%)

(b). Find the mean and the variance of the random variable Y;　(5%)

(c). If random variable Y is now obtained from $Y = -2X + 3$, find the mean of Y, its variance, and its pdf.　(5%)

4. A communication system transmits binary information over a channel that introduces random bit error with probability $p = 10^{-3}$. The transmitter transmits each information bit three times:

$$0 \longrightarrow 000; \qquad 1 \longrightarrow 111,$$

and a decoder takes a majority vote of the received bits to decide on what the transmitted bit was. Find the probability that the receiver will make an incorrect decision.　(10%)


（背面仍有題目，請繼續作答）

2312 B
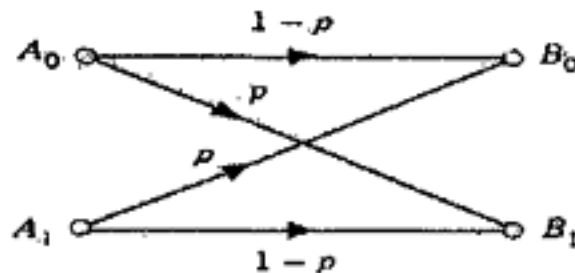
5. Let A be an n×n matrix, det(A) denote the determinant of A and adj(A)

denote the adjoint of A.
(a) Prove that A is invertible if and only if adj(A) is invertible. (7%)
(b) Prove that $\det(\text{adj}(A)) = (\det(A))^{n-1}$. (8%)

6. Let A= $\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & -1 & 2 \\ 2 & 5 & 1 & 4 \\ 1 & 1 & 2 & -1 \end{bmatrix}$

(a) Find a basis for the column space of A.   (5%)
(b) Find an orthogonal basis for the null space of A.   (10%)

7. The line spectrum, in volts, of a certain periodic function f(t) is shown below.
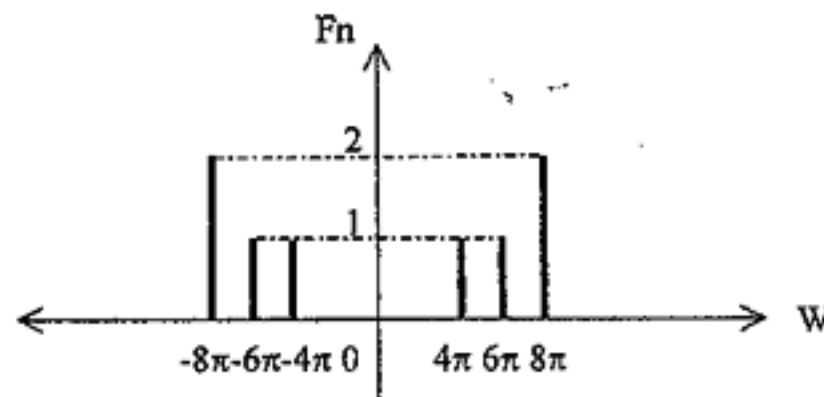
(a) What is the value of the period, T?   (2%)
(b) What is the average power of f(t) when f(t) is applied to a 1-Ohm resistance.   (3%)

(c) Let $g(e) = f\left(\dfrac{t}{2}\right)$, Sketch the line spectrum of g(t).   (3%)

8. Find the convolution integral for f(t) and h(t), where f(t)=A sin πt u(t) and h(t) is shown below.   (12%)