

1. Consider the unity feedback system with  $G(s) = K / [(s+3)(s+5)]$ .
- (a) Show that this system cannot operate with a settling time of 2/3 second and a percent overshoot of 1.5% with a simple gain adjustment. (10%)
- (b) Design a lead-compensator so that the system meets the transient response characteristics of part (a). Specify the compensator's pole, zero, and the required gain. (10%)

2. The closed-loop transfer function of a control system is

$$T(s) = \frac{s^2 + K_1 s + K_2}{s^4 + K_1 s^3 + K_2 s^2 + 5s + 1}$$

Determine the range of  $K_1$  in order for the system to be stable. What is the relationship between  $K_1$  and  $K_2$  for stability? (15%)

3. Consider a digital system described by

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned}$$

For the case that  $u(k)$  is not zero, derive the conditions for observability. (15%)

4. The dynamics of an armature controlled DC motor are described by the following equations

$$\begin{aligned} \frac{d\theta}{dt} &= \omega \\ J \frac{d\omega}{dt} &= K_a i_a \\ L \frac{di_a}{dt} &= -Ri_a - K_b \omega + v_a \end{aligned}$$

where  $\theta$  is the angular displacement,  $\omega$  is the speed,  $i_a$  is the armature current,  $J$  is the armature moment of inertia,  $R$  is the armature winding resistance,  $L$  is the armature inductance,  $K_a$  and  $K_b$  are constants. The control input is the voltage  $v_a$  and it is assumed that the angular speed is the output. Is the system controllable? Is the system observable? (16%)

5. Find the state transition matrix for  $A(t) = \begin{bmatrix} -1 & \cos t \\ 0 & 0 \end{bmatrix}$ . (10%)

6. Answer the following short questions

- a. What is the significance of the sensitivity function in feedback control? (6%)
- b. A compensator is described by  $G_c(s) = \frac{1 + \alpha\tau s}{\alpha(1 + \tau s)}$  for some  $\alpha$  and  $\tau$ . What is the maximal phase angle of the compensator as a function of  $\alpha$ ? (6%)
- c. What is a nonminimum phase system? What are the difficulties in feedback control design if the open-loop system is of nonminimum phase? (6%)
- d. Given the Bode diagram of the open loop system (in a unity feedback configuration), what feedback information can you deduce? (6%)