1. Find the Fourier cosine series of the function $f(x) = x^2,\ x \in (0,1)$ (10%)

   From the series, find $\sum_{n=1}^{\infty} \frac{1}{n^2}$. (5%)

2. Let the random variables X and Y have the joint pdf.

   $$f(x,y) = x + y, \quad 0 < x < 1, 0 < y < 1$$
   $$= 0, \quad elsewhere$$

   (a) Find the pdf of X. (5%)

   (b) Find the correlation coefficient of X and Y. (5%)

3. The pdf of Pareto distribution X is $f(x) = ax^{-(a+1)},\ 1 \le x \le \infty$, where a is a positive shape parameter. If $1 < a < 2$,

   (a) find the mean of Pareto distribution (2%)

   (b) find the variance of Pareto distribution. (4%)

   (c) find the pdf of X^3. (4%)

4. $X_1, X_2, \cdots, X_n$ are independent and each is uniformly distributed in the interval (0,1).

   (a) Find the pdf of $Y = \max(X_1, X_2, \cdots, X_n)$.(5%)

   (b) Find the pdf of $Z = |X_1 - X_2|$ (5%)

   (c) Find the pdf of W, if $X_1 = 1 - e^{-W/\lambda}$ (5%)

5. **(a)** If the discrete Fourier transform (DFT) for an N-point sequence $x[n]$ is defined as $X[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi kn}{N}}$, please give the formulation of the inverse DFT and justify it. (5%)

(b) An 8-point sequence is given as $y[n] = [-2\ \ 4\ \ -4\ \ 7\ \ -2\ \ 4\ \ -4\ \ 7]$. If $Y[k]$ denotes the 8-point DFT (*defined in 5.(a)*) of $y[n]$, please compute $\sum_{k=2}^{9} (-1)^k Y[k]$. (5%)

6. If an $N \times N$ matrix $\boldsymbol{A}$, whose elements are expressed as $\{a_{ij}$, for $1 \le i, j \le N\}$, has eigenvalues, $\lambda_1, \lambda_2, \ldots \lambda_N$, Prove that,

(a) $\sum_{i=1}^{N} \lambda_i = \sum_{i=1}^{N} a_{ii}$. (5%)

(b) $det[A] = \lambda_1 \lambda_2 \ldots \lambda_N$. (5%)

7. If $F(s) = \frac{s^2 - 6s + 6}{(s^2 - 4s + 4)^2}$ denotes the Laplace transform of $f(t)$, please find

(a) $f(0) = ?$ (5%)

(b) $\int_0^{\infty} f(t)dt = ?$ (5%)

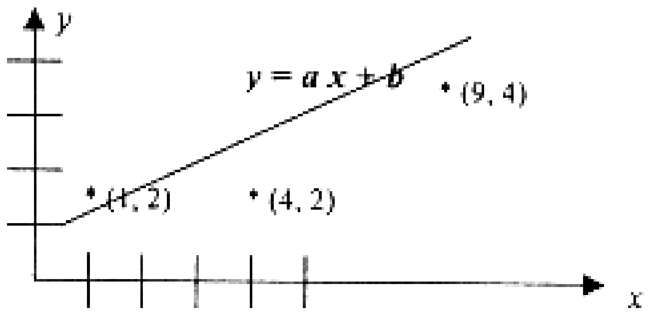
8. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0.8 & 2 \\ 0 & 0 & 0.5 \end{bmatrix}$, please compute *(All results expressed in F5.1 × 10^m, where m is an integer)*

(a) $A^6 = ?$ (2%)

(b) All the eigenvalues of A^6 (3%)

(c) All the eigenvalues of A^{112} (5%)

9. **(a)** After three experiments, we obtain (1, 2), (4, 2), and (9, 4) points. To fit all these four points to one line, $y = ax + b$ in the least mean square error sense, please find optimal a and b. (5%)



(b) Generalize the above problem for 50 experiments. $(x_1, y_1), (x_2, y_2), \ldots, (x_{50}, y_{50})$.(5%)