1. Construct a minimum states finite-state machine that determines whether the input string has a 1 in the last position and a 0 in the third to the last position read so far. (12%)

2. (a) Let $G=(V, E)$ be a connected bipartite undirected graph with V partitioned as $V_1 \cup V_2$. Prove that if $|V_1| \neq |V_2|$, then G cannot have a Hamilton cycle. (6%)
(b) Give an example of a connected bipartite undirected graph $G=(V, E)$, where V is partitioned as $V_1 \cup V_2$ and $|V_1| = |V_2| - 1$, but G has no Hamilton path. (6%)

3. If $G=(V, E)$ is an undirected graph, any subgraph of G that is a complete graph is called a clique in G. The number of vertices in a largest clique in G is called the clique number for G and is denoted by $\omega(G)$.
(a) How are $\chi(G)$ and $\omega(G)$ related? (7%)
(b) Is there any relationship between $\omega(G)$ and $\beta(\overline{G})$? (6%)
($\beta(G)$: the independence number of G is the size of a largest independent set in G. $\chi(G)$: the chromatic number of G is the minimum number of colors needs to properly color G.)

4. In how many ways can Nicole paint the eight regions of the square shown in Fig.1 if she actually uses exactly four of the five available colors? (13%)

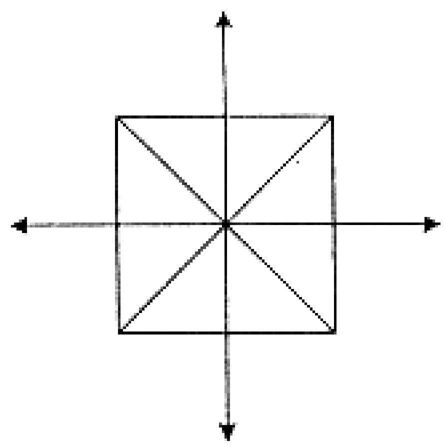

Fig.1

(背面仍有題目,請繼續作答)

5. Let f and g be two Boolean functions of n variables. Function f and function g are said to be equivalent if f can be obtained by permuting and/or complementing the Boolean variables of g, i.e.,

   $f(x_1, x_2, \cdots, x_n) = g(x_{\alpha(1)}^{c_1}, x_{\alpha(2)}^{c_2}, \cdots, x_{\alpha(n)}^{c_n})$, where $x_{\alpha(i)}^{c_i} = x_{\alpha(i)}$ or $\bar{x}_{\alpha(i)}$ and α is a permutation on $\{1,2,\ldots,n\}$.

   Let VPC be the set of all variable permutation and complementation transformations on n variables. Show that VPC forms a group. (17%)

6. There are seven people A, B, C, D, E, F, G in an international conference. A speaks English; B speaks Chinese and English; C speaks English, Italian and Russian. D speaks Chinese and Japanese. E speaks Italian and German. F speaks Russian, Japanese and France. G speaks German and France. Can their seats be arranged as a circle such that every one can chat with his two neighbors? Explain your point. (16%)

7. Try to solve the recurrence relation for the Fibonacci sequence, where it is defined as: $F_n=F_{n-1}+F_{n-2}$, where $F_0=F_1=1$. (17%)