1. Let **D** be the differentiation operator on $\mathbf{P}_3$ (where $\mathbf{P}_n$ denotes the set of all polynomials of degree less than n) and let $\mathbf{A} = \{p \in \mathbf{P}_3 \mid p(0) = 0\}$. Please show that **D** maps $\mathbf{P}_3$ onto $\mathbf{P}_2$, but is not one-to-one. (10%)

2. Please show that the eigenvectors of a Hermitian matrix belonging to distinct eigenvalues are orthogonal. (10%)

3. Let $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 8 \end{bmatrix}$, please find $\mathbf{A}^{\frac{1}{3}}$. (10%)

4. Find the general solutions of the given differential equations.
   (a) $y''+5y' = xe^{-x}\sin(3x)$. (10%)
   (b) $(2x^2+3x+1)y''+2xy'-2y=0$; $y_1(x)=x$ is a solution for x in any interval not containing -1 or $-1/2$. (10%)

5. Use the Laplace transform to solve the system. (15%)
   $x'+2x-y'=0,\ x'+y+x=t^2;\ x(0)=y(0)=0$.

6. (a) Find the Fourier integral representation of the function

$$f(t)=\begin{cases} 0 & -\infty < t \le -1 \\ 1+t & -1 \le t \le 0 \\ 1-t & 0 \le t \le 1 \\ 0 & 1 \le t < \infty \end{cases}$$

   and express the integral which approximates this function for frequencies between 0 and ω_0 in terms of the sine-integral function. (15%)

   (b) Use the result of (a), show that

$$\frac{\pi}{2} = \int_0^\infty \frac{1-\cos\omega}{\omega^2}d\omega. \quad (5\%)$$

7. Evaluate $\oint_c (\bar{z}-a)^{-1}(b-\bar{z})^{-1}(z^2+z^{-2})dz$, $0<|a|<r<|b|$, where c is the positive orientation of the circle $\{z; |z|=r\}$. (15%)