

1. A dynamic vibration absorber is shown in Fig. 1.
 (a) Obtain the differential equations describing the system in which $F(t)$ is the input and $y_1(t)$ is the output. (8%)
 (b) Determine the values of M_2 and k_{12} such that the main mass M_1 does not vibrate when $F(t) = A \sin \omega_0 t$. (7%)

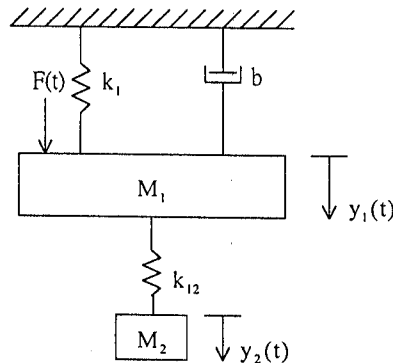


Fig. 1

2. Consider the feedback system in Fig. 2. Fig. 3 depicts the Bode diagram of $G(s)$. Determine an approximate second-order model of $G(s)$. (10%)

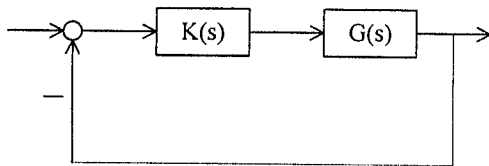


Fig. 2

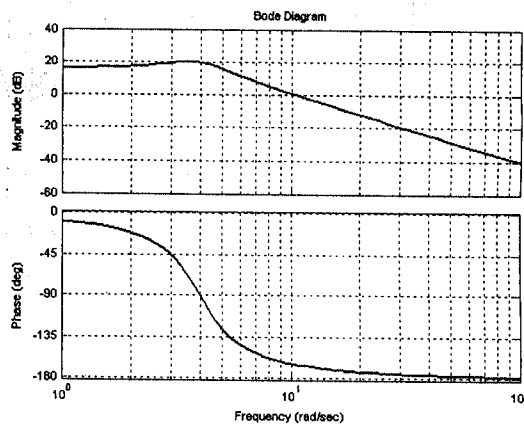


Fig. 3

3. With respect to Problem 2. Design a feedback controller $K(s)$ such that the closed-loop system is stable and has at least 40° phase margin and 10 dB gain margin. (10%)
4. Prove that if A is a stability matrix, then the Lyapunov equation $A^T P + PA + Q = 0$ has a unique solution for every Q . (Note: A^T is the transpose of A). (15%)
5. For a discrete-time system with $T = 0.01$ sec, the characteristic equation is given by $(z - 0.99)(z^2 - 0.5z + 1.0) = 0$.
 (a) Show that this system is marginally stable. (8%)
 (b) Find the frequency at which the system will oscillate. (7%)
6. (a) Name five representations of systems in state space. (5%)
 (b) Why are marginally stable systems considered unstable under the BIBO definition of stability? (5%)
 (c) Define *system type*. (5%)
 (d) Describe the conditions that must exist for all closed-loop poles and zeros in order to make a 2nd-order approximation. (5%)
 (e) Why is there more improvement in steady-state error if a PI controller is used instead of a lag network? (5%)
 (f) Describe the change in the open-loop frequency response magnitude plot if time delay is added to the plant. (5%)
 (g) Briefly explain how a lag network allows the low-frequency gain to be increased to improve steady-state error without having the system become unstable. (5%)