1. The coefficient of the quadratic equation $ax^2 + bx + c = 0$ are determined by tossing a fair die three times (the first outcome is a, the second one is b, and the third one c). Find the probability that the equation has no real roots. (10%)

2. Suppose that, from a box containing D defective and $N - D$ nondefective items, n ($\leq D$) are drawn one-by-one, at random and without replacement. Prove that the probability Pr(the kth item drawn is defective) equals D/N. (15%)

3. Let Z be a continuous random variable with density function f. Find the density functions of $X = |Z|$ and $Y = Z^2$. (10%)

4. On the days Linda walks from home to work, she has to cross the street at a certain point. When, at that point, Linda needs a gap of 15 seconds in the traffic to cross the street. Suppose that the traffic flow at that point is a Poisson process, and the mean time between two consecutive cars passing by Linda is 7 seconds. Find the expected value of the time Linda has to wait before she can cross the street. (15%)

5. (a) If the inverse discrete Fourier transform (DFT) for an N-point sequence $x[n]$ is defined as $x[n] = \sum_{k=0}^{N-1} X[k] e^{+j\frac{2\pi kn}{N}}$, please give the formulation of the DFT and prove its correctness. (5%)

 (b) An 8-point sequence is given as $x[n] = [-2\ \ 3\ -5\ \ 2\ -1\ \ 3\ -5\ \ 6]$. If $X[k]$ denotes the 8-point DFT (*defined in 5.(a)*) of $x[n]$, please compute **(i)** $\sum_{k=0}^{7} X[k]$; **(ii)** $X[8]$.. (5%)

6. $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1.2 & 2.1 \\ 2.9 & 4.3 \\ 5.2 & 6.1 \\ 6.8 & 8.1 \end{bmatrix}$, please find $\mathbf{Q}$ to minimize $|\mathbf{A}-\mathbf{BQ}|^2$, where $\mathbf{Q}$ is an orthogonal matrix and $|\bullet|^2$ denotes the sum of square elements of the matrix. (10%)

7. If $v(t) = t^2 + \sin(2\pi f_0 t)$, please find: (a) $\int_{-\infty}^{\infty} [v(t)-1]\delta(t)dt = ?$ (5%)

 (b) $\int_{1}^{100} v(t)\delta(t+4)dt = ?$ (5%)

(背面仍有題目,請繼續作答)

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8. An n x n symmetrical tridiagonal Toeplitz matrix has the form

$$A=\begin{bmatrix} b & a & \cdots & & 0 \\ a & b & a & & \vdots \\ 0 & a & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & a \\ 0 & \cdots & 0 & a & b \end{bmatrix},$$

please show that it has eigenvalues $\lambda_k = a + 2b\cos(k\pi / n+1)$ for $k = 1$ to n and find their corresponding eigenvectors. (10%)

9. The desired signal is given as

$$x(t)=\begin{cases} 1 & \text{for } 0 \le t < 1/2 \\ 2 & \text{for } 1/2 \le t < 1 \end{cases}$$

For given three basis signals $\phi_1(t)=1$, $\phi_2(t)=t$, and $\phi_3(t)=t^2$ and the approximating signal is given by $\hat{x}(t)=c_1\phi_1(t)+c_2\phi_2(t)+c_3\phi_3(t)$, please find the optimal c_1, c_2, and c_3 in sense of minimizing the mean square error criterion (10%)

$$\xi=\int_0^1 \left| x(t)-\sum_{k=1}^{3} c_3\phi_3(t) \right|^2 dt.$$