(1).

(a). Determine the finite Fourier sine transform $F_S(n) = \int_0^{\pi} f(x)\sin(nx)dx$,

$n = 1, 2, 3, \ldots$, for the function $f(x) = x^2$, $0 \le x \le \pi$. (7%)

(b). For a given function $d(t) = \sum_{n=-\infty}^{\infty} \delta(t-3n)$, graph the convolution

$g(t)*d(t)$, where

$$g(t) = \begin{cases} t & \text{if } 0 \le t \le 2, \\ 4-t & \text{if } 2 \le t \le 4, \\ 0 & \text{if } t < 0 \text{ or } t > 4. \end{cases}$$ (8%)

(2). If the random variable X is Gaussian with probability density function (pdf) $f_X(x) = (1/(2\pi)^{1/2}\sigma_X)\exp[-(x-\mu_X)^2/2\sigma_X^2]$, then the random variable $Y = \exp(X)$ is the lognormal random variable with pdf $f_Y(y) = f_X(\ln y)/y$.

(a). Show that the moments about the origin for the lognormal random variable Y are

$$E[Y^n] = \exp(n\mu_X + n^2\sigma_X^2/2).$$ (10%)

(b). Using the above, show that the mean and the variance of the lognormal random variable Y are

$$\mu_Y = \exp(\mu_X + \sigma_X^2/2);$$
$$\sigma_Y^2 = \exp(2\mu_X + \sigma_X^2)[\exp(\sigma_X^2) - 1].$$ (10%)

(3). The random variable **X** is Rayleigh, and has the pdf

$$F_X(x) = (2x/7)\exp(-x^2/7)u(x).$$

If 20 independent observations of **X** are summed, then make an estimate,

by using the central limit theorem, the probability of the sum being between 40 and 50. (15%)

(4).

For matrix $A = \begin{bmatrix} 3 & -2 & -5 \\ 4 & -1 & -5 \\ -2 & -1 & -3 \end{bmatrix}$, find A^{-1} and A^4. (10%)

(背面仍有題目,請繼續作答)

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(5).

If $A \in M_n$ is Hermitian and $A \neq 0$, show that

$$rank(A) \geq [tr(A)]^2 / [tr(A^2)],$$

where $tr(A)$ denotes the trace of A. (10%)

(6).

Solve $y''+2y'+5y = e^{-t}\sin t$, $y(0)=0, y'(0)=1$ by using Laplace Transform.

(10%)

(7).

Find the Laplace transforms of the following periodic function f(t) and aperiodic function g(t). (20%)