

1. Find the general solutions of the given differential equations.

(a)  $(y^2 + 1)dx = y \sec^2 x dy$ . (10%)

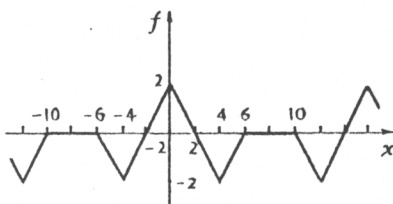
(b)  $y'' - y' - 12y = 2 \sinh^2 x$ . (10%)

2. Given that  $t(1-t)y'' + 2y' + 2y = 6t$ ;  $y(0) = 0, y(2) = 0$

(a) Identify the type of the problem for the given equation and conditions. (5%)

(b) Please use the Laplace transform to solve the problem. (15%)

3. Find the Fourier series of the periodic function  $f$ , the graph of which is shown in the following figure. (15%)



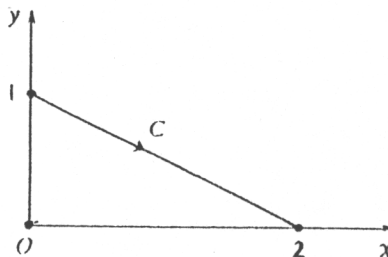
4. Knowing that the ML bound describes

$$\left| \int_C f(z) dz \right| \leq ML$$

where  $|f(z)| \leq M$  on  $C$  and  $L$  is the length of  $C$ . Find the ML bound of the following integral

$$I = \int_C \frac{e^z}{z^2} dz$$

where  $C$  is the straight line as shown. (15%)



5. Please find the least squares solution to the system described by  $x_1 + x_2 = 3$ ,  $-2x_1 + 3x_2 = 1$ , and  $2x_1 - x_2 = 2$ . (15%)

6. Let  $\lambda$  an eigenvalue of an  $n \times n$  matrix  $B$ , and let  $X$  be an eigenvector belonging to  $\lambda$ . Please show that  $e^\lambda$  is an eigenvalue of  $e^B$  and  $X$  is an eigenvector of  $e^B$  belonging to  $e^\lambda$ . (15%)