1. A feedback control system has a characteristic equation

$$s^3+(1+K)s^2+15s+(4+12K)=0$$

The parameter K must be positive. What is the maximum value K can assume before the system becomes unstable? When K is equal to the maximum value, the system oscillates. Determine the frequency of oscillation. (20%)

2. A unity feedback system with $G(s)=\dfrac{K}{(s+1)(s+2)(s+4)(s+5)}$ is operating with a dominant pole damping ratio of 0.707. Design a PD controller so that the settling time is reduced by a factor of 2. (20%)

3. Determine the stability of the system $T(z)=\dfrac{z+1}{z^3+3.7z^2-1.3z-0.4}$. (10%)

4. In comparison with a continuous-time control system, a digital control system is subject to sampling effects and quantization effects. Explain how the quantization effects affect the selection of the sampling rate. (10%)

5. A linear time-invariant system can be modeled in terms of state space representation: $\dot{x}=Ax+bu$ and $y=c^Tx$ in which x is the state, y is the output, and u is the input. Suppose that the transfer function of the system can be described by $H(s)=c^T(sI-A)^{-1}b=h_0+h_1s^{-1}+h_2s^{-2}+\cdots$. Find a method to determine the state space realization (A,b,c^T) in terms of $h_0,h_1,h_2,\cdots$. (20%)

6. In plotting the root locus of a negative feedback control system, it is known that as the locus approaches infinity, the root locus approaches straight lines as asymptotes. Let n and m be the numbers of finite poles and zeros of the open-loop system, respectively. Show that the equation of the asymptotes is given by the real-axis intercept σ_a and angle θ_a as follows:

$$\sigma_a=\frac{\sum \text{finite poles}-\sum \text{finite zeros}}{n-m}$$

and

$$\theta_a=\frac{(2k+1)\pi}{n-m}$$

where $k=0,1,2,\cdots$ and the angle is given in radians with respect to the positive extension of the real-axis. (20%)