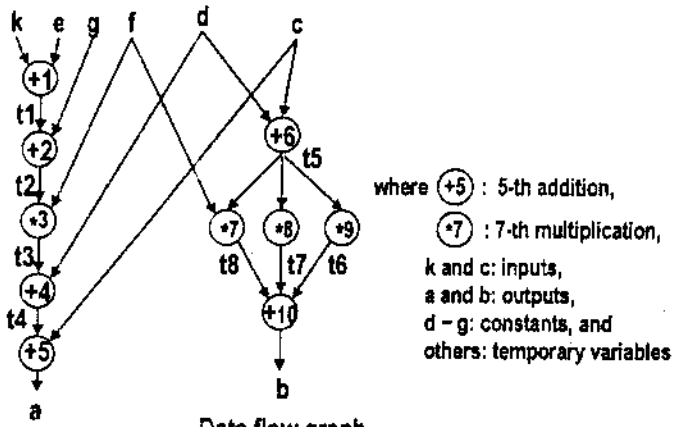
1. Please derive a fastest scheduling (i.e., a minimum state diagram) and the minimum number of registers used in the scheduling for the following data flow graph using 1 adder and 1 multiplier with a graph theoretic approach. Assume both the adder and the multiplier have one clock cycle delay. (Hint : use graph coloring) (25%)



Data flow graph

2. a). Let g be a function, and B_1 and B_2 be two sets, what is the relationship between $g(B_1 \cap B_2)$ and $g(B_1) \cap g(B_2)$? b). Please prove the relationship in a). (13%)

3. Let $n, r, s \in \mathbf{Z}^+$ with $n, r, s \geq 2$, $n = rs$, and $\gcd(r, s) = 1$. If f: $\mathbf{Z}_n \rightarrow \mathbf{Z}_r \times \mathbf{Z}_s$ is a ring isomorphism with $f(a) = (1, 0)$ and $f(b) = (0, 1)$, please compute $f^{-1}(m, t) = ?$ where $(m, t) \in \mathbf{Z}_r \times \mathbf{Z}_s$. (12%)

4. The Fibonacci sequence is defined as: $F_n = F_{n-1} + F_{n-2}$, where $F_0 = F_1 = 1$. We have two ways to find out the n-th Fibonacci number, i.e., recursively and iteratively. Show the time complexities for the recursive and iterative algorithms, respectively. What's your comment about the results ? (17%)

5. Let $d_1, d_2, \ldots, d_n$ be n positive integers, $n \geq 2$ and $\sum_{i=1}^{n} d_i = 2n - 2$. Show that there exists a tree with node degree $d_1, d_2, \ldots, d_n$. (16%)

6. Let $y = x_1 \oplus x_2 \oplus x_3 \oplus \ldots \oplus x_n$, $n \geq 2$. How many logic gates in Big-O notation do you need to implement y by using 2-level logic? Give a better solution and shows the gate number complexity, in Big-O notation, too. (Hint : $\oplus$ denotes exclusive-OR function) (17%)