1. Find the general solution of the equation (10%)
$x^2y''-2xy'+2y=\ln(x)+1$.

2. Use the Laplace transform to solve the system (10%)
$y_1'-2y_2'+3y_3=0,\ y_1-4y_2'+3y_3'=t,\ y_1-2y_2'+3y_3'=1;$
$y_1(0)=y_2(0)=y_3(0)=0.$

3. Given the equation $16x^2y''-4x^2y'+3y=0$,
   (a) Show that zero is a regular singular point. (3%)
   (b) Find and solve the indicial equation. (4%)
   (c) Determine the recurrence relation. (4%)
   (d) Find the first four nonzero terms of two linearly independent Frobenius solutions. (4%)

4. Fill in the blank or answer True or False (10%)
   (a) If f is not piecewise continuous on $[0,\infty)$, then the Laplace transform $L\{f(t)\}$ will not exist.____
   (b) $F(s)=\dfrac{s^2}{s^2+4}$ is not the Laplace transform of a function that is piecewise continuous and o exponential order.________
   (c) $L^{-1}\left\{\dfrac{s}{s^2-10s+29}\right\}=$ ________
   (d) $L^{-1}\left\{\dfrac{s+\pi}{s^2+\pi^2}e^{-s}\right\}=$ ________. (Note: $u(t-k)=\begin{cases}1, & t\ge k\\ 0, & t<k\end{cases}$ is a unit step function)
   (e) If $L\{f(t)\}=F(s)$ and $k>0$, then $L\{e^{a(t-k)}f(t-k)u(t-k)\}=$ ________________.(Note $u(t-k)=\begin{cases}1, & t\ge k\\ 0, & t<k\end{cases}$ is a unit step function)

5. Fill in the blank or answer True or False (10%)
   (a) The functions $f(x)=x^2-1$ and $f(x)=x^5$ are orthogonal on $[-\pi,\pi]$.________
   (b) To expand $f(x)=|x|+1$, $-\pi<x<\pi$, in an appropriate Fourier series we would use a series.
   (c) The Fourier series of $f(x)=\begin{cases}3, & -\pi,x<0\\ 0, & 0\le x<\pi\end{cases}$ will converge to ________at $x=0$.
   (d) $y=0$ is never an eigenfunction of a Sturm-Liouville problem.________
   (e) $\lambda=0$ is never an eigenvalue of a Sturm-Liouville problem.________

(背面仍有題目,請繼續作答)

6. Fill in the blank or answer True or False (10%)

(a) The sector defined by $-\pi/6 < \arg z < \pi/6$ is a simply connected domain._______

(b) The value of $\int_C \frac{z-2}{z}dz$ is the same for any path C in the right half-plane $\text{Re}(z) > 0$ betwee

$z = 1+i$ and $z = 10+8i$.________

(c) If $f(z) = z^3 + e^z$ and C is a contour $z = 8e^{it}$, $0 \le t \le 2\pi$, then

$\oint_C \frac{f(z)}{(z+\pi i)^3}dz =$ ________

(d) $\oint \frac{1}{(z-z_0)(z-z_1)}dz = 0$ for every simple closed contour C that enclosed the points z_0 and z_1._

(e) If $|f(z)| \le 2$ on $|z|=3$, then $\left|\oint_C f(z)dz\right| \le$________.

7. Let $Q(X) = 3x_1^2 + 6x_2^2 + 3x_3^2 - 4x_1x_2 + 8x_1x_3 + 4x_2x_3 = 0$.

(a) Find a unit vector X in $\Re^3$ at which $Q(X)$ is maximized, subject to $X^T X = 1$, and th maximum of $Q(X)$. (Hint: Two eigenvalues of the matrix of the form are 7 and -2.) (10 %)

(b) Find the solution set, not just one particular solution, for Part (a). (10 %)

8. Find a singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}$. (15 %)