1. For a system with open-loop transfer function

$$G(s) = \frac{10}{s[(s/1.4)+1][(s/3)+1]},$$

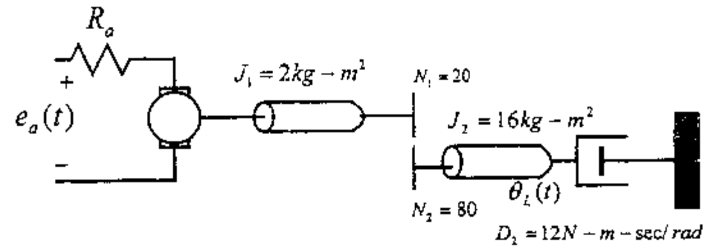
design a lag compensator with unity DC gain so that PM $\geqq 40^\circ$. What is the approximate bandwidth of this system? **(25%)**

2. The open-loop plant of a unity feedback system has the transfer function

$$G(s) = \frac{1}{s(s+2)}.$$

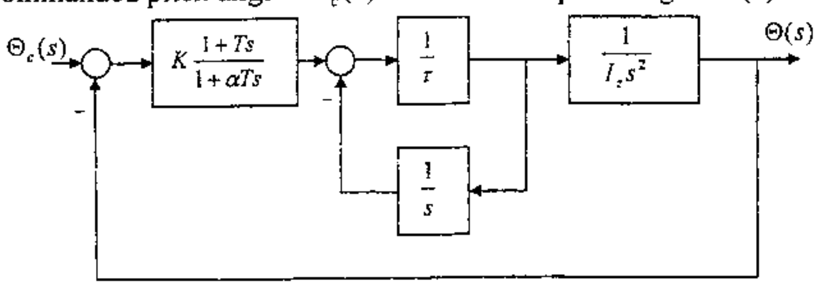
Determine the transfer function of the equivalent digital plant using a sampling period of $T = 1$ sec, and design a proportional controller for the discrete-time system that yields dominant closed-loop poles with a damping ratio ζ of 0.7. **(25%)**

3. The motor in the figure has the following characteristics for an applied voltage of 100 V. When the speed is 200 (rad/sec), the torque is 40 (N-m). When the speed is 100 (rad/sec), the torque becomes 60 (N-m). Determine the transfer function $G(s) = \dfrac{\theta_L(s)}{E_a(s)}$. **(15%)**



4. A linear time-invariant system is modeled in terms of the state space representation: $\dot{x}(t) = Ax(t)$ in which x is the state. Suppose that there exists a positive definite matrix P such that $A^T P + PA + Q = 0$ for a positive semi-definite matrix Q, the system is thus exponentially stable, that is, there exist positive γ and k such that the norm of $x(t)$ is bounded: $\|x(t)\| \le ke^{-\gamma t}\|x(0)\|$. Determine γ and k in terms of P and Q. (Note: A^T is the transpose of the matrix A). **(15%)**

5. The block diagram for a pitch axis attitude control system is depicted in the figure. Determine the transfer function from the commanded pitch angle $\Theta_c(s)$ to the actual pitch angle $\Theta(s)$. **(10%)**



6. Suppose that in the figure of the previous problem, $I_z = 5000$ in-lb-sec^2, $\tau = 20$ sec, $T = 5$ sec, and $K = 200$. Determine the range of α so that the closed-loop system is stable. **(10%)**