1. (20%) The state-space representation of the transfer function

$$G(s)=\frac{Y(s)}{U(s)}=\frac{s+2}{s^3+8s^2+24s+32}$$ has the following form

$$\dot{x}=\begin{bmatrix}0 & 2 & 0\\ 0 & 0 & 2\\ a & b & c\end{bmatrix}x+\begin{bmatrix}0\\0\\1\end{bmatrix}u,\quad y=\begin{bmatrix}d & e & f\end{bmatrix}x.$$ Determine $a,b,c,d,e,$ and f.

2. (30%) With respect to the system in Figure 1, answer the following questions. (Assume that $K\geq 0$)

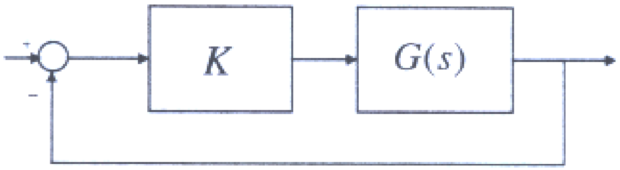

Figure 1.

(i) When $G(s)=\dfrac{s^2+4s+4}{s^5+8s^4+12s^3+30s^2+22s+28}$, determine the angles and the real-axis intercept of the asymptotes of the root loci when K is sufficiently large.

(ii) When $G(s)=\dfrac{s+2}{s(s+1)(s^2+2s+2)}$, determine the angle of departure of the root loci at $-1+j1$.

(iii) Same as (ii), calculate the frequency of $j\omega$-axis crossing for the root loci.

(iv) Same as (ii), find the value of K so that the steady state error is minimized.

(v) Same as (ii), determine the sensitivity of the steady-state error to changes in parameter K.

3. (12%, 8%) A system is shown in Figure 1, where $G(s)=(s+5)^2/s^3$. (a) Please determine the range of K via the Nyquist plot such that the controlled system is stable. (b) If K=20, please give the gain margin of this system and its meaning.

4. (10%) Compute $\sin At$ with $A=\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}$.

5. (20%) Consider the state equation

$$\dot{X}=\begin{bmatrix}\delta & 0 & 0 & 0 & 0\\ 0 & \alpha & \beta & 0 & 0\\ 0 & -\beta & \alpha & 0 & 0\\ 0 & 0 & 0 & \gamma & \eta\\ 0 & 0 & 0 & -\eta & \gamma\end{bmatrix}X+\begin{bmatrix}b_1\\ b_2\\ b_3\\ b_4\\ b_5\end{bmatrix}u,\qquad y=\begin{bmatrix}c_1 & c_2 & c_3 & c_4 & c_5\end{bmatrix}X$$

Suppose δ, α, β, γ and η are all distinct nonzero real constants. Please find the conditions of b_i and c_i such that the system is controllable and observable.