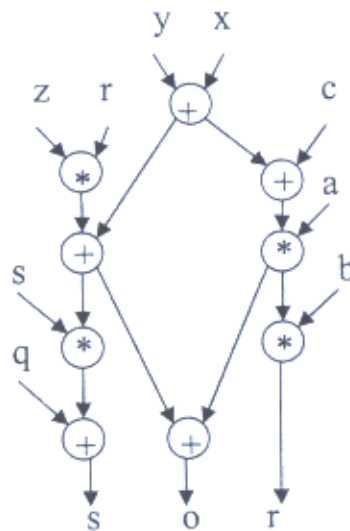


- Solve the following recurrence relations. (15%)

$$a_{n+2}^2 - 5a_{n+1}^2 + 6a_n^2 = 7n, \quad n \geq 0, \quad a_0 = a_1 = 1$$

- Please derive the minimum state diagram of a mealy-style clocked sequential circuit that recognizes the input sequence consisting of exactly two zeros followed by a 10. (15%)
- Please derive a fastest scheduling (i.e., a minimum state diagram) and the minimum number of registers used in the scheduling for the following data flow graph using 1 adder and 1 multiplier with a graph theoretic approach. Assume both the adder and the multiplier have one clock cycle delay. (Hint : use graph coloring) (15%)



- Let  $(A, R)$  be a poset in which the length of longest chain is  $n$ . Use mathematical induction to prove that the elements of  $A$  can be partitioned into  $n$  antichains  $C_1, C_2, \dots, C_n$  (where  $C_i \cap C_j = \emptyset$ , for  $1 \leq i < j \leq n$ ).  
Definition of antichain: let  $(A, R)$  be a poset, and let  $\emptyset \neq C \subseteq A$ . If  $(C \times C) \cap R = \emptyset$ , then for all distinct  $x, y \in C$  we have  $x \not\preceq y$  and  $y \not\preceq x$ . The elements of  $C$  are said to form an antichain in the poset  $(A, R)$ . (10%)
- (a) Find a graph  $G$  where both  $G$  and  $\overline{G}$  are connected.  
(b) If  $G$  is a graph on  $n$  vertices, for  $n \geq 2$ , and  $G$  is not connected, prove that  $\overline{G}$  is connected. (20%)
- Find the generating functions for the sequence  $1, -2, 3, -4, 5, -6, 7, \dots$  (10%)
- How many ways can we have to pattern 2 colors in the vertices of a cube? (15%)