1. Find the general solution of the following ordinary differential equation

$$y' + \frac{2x-5y-9}{4x-y-9} = 0 \qquad (17\%)$$

2. Solve the initial value problem

$$y'' + 9y = f(t); \qquad y(0) = y'(0) = 1; \qquad (18\%)$$

$f(t)=0$ if $0 \le t < \pi$, $f(t)=\cos(t)$ if $t \ge \pi$.

3. $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 27 & -27 & 9 \end{bmatrix}$

(a) Find the eigenvalues of A. (4%)

(b) Is A diagonalizable? Justify your answer. (6%)

(c). Compute e^{tA}, (10%)

where $e^{tA} = I + \frac{t}{1!}A + \frac{t^2}{2!}A^2 + \frac{t^3}{3!}A^3 + \ldots + \frac{t^n}{n!}A^n + \ldots$

4. Compute the complex Fourier series of $f(x)$. Note that $f(x)$ has period 2 ,

and $f(x) = \begin{cases} x^2 + x & \text{for} \quad 0 \le x < 1 \\ x^2 - x + 2 & \text{when} \quad 1 \le x < 2 \end{cases}$ (15%)

5. Evaluate $\int_0^{\pi} \frac{d\theta}{(a+\cos\theta)^2}$ $\quad a>1.$ (15%)

6. Given that $\oint_C \frac{f(z)}{z} dz = 2\pi i$ and $\oint_C \frac{f(z)}{(z-1)^2} dz = 4\pi i$.

If let $f(z) = a + bz$, find the a and b. (15%)