

1. (18%) Given the following systems (a)  $\frac{16}{s^2 + 4s + 16}$ , (b)  $\frac{16}{s^2 + 6s + 16}$ , (c)  $\frac{36}{s^2 + 6s + 36}$ , (d)  $\frac{16(s+1)}{s^2 + 4s + 16}$ , (e)  $\frac{16}{(s+1)(s^2 + 4s + 16)}$ , answer the following questions (may be multiple choices).

- Which systems (among a, b, c) have the largest percent overshoot with respect to step input?
- Which system (among a, d, e) has the shortest rise time with respect to step input?
- Which systems (among a, b, c) have the shortest settling time?
- What is the percent overshoot of the system (b)?
- What is the damped frequency of oscillation of the system (c)?
- What is the peak time of the system (a)?

2. (14%) The linear model of a phase-lock loop is depicted in Figure 1. The loop filter is given by  $F(s) = \frac{10(s+4)}{(s+1)(s+40)}$ . Let  $K_v = K_a K$ . Sketch the root locus of the system as a function of  $K_v$ . Specify important parameters pertaining to the root locus.

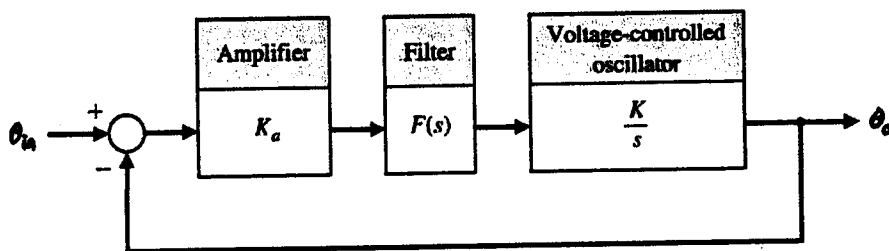


Figure 1.

3. (16%) A lead compensator has the form  $G_c(s) = \frac{1 + \alpha\tau s}{\alpha(1 + \tau s)}$  for some  $\tau$  and  $\alpha > 1$ .

Suppose that at least  $30^\circ$  phase angle is desired, what is the minimum value of  $\alpha$ ?

(背面仍有題目,請繼續作答)

4. (16%) A feedback control system has a characteristic equation  $s^3 + (2 + \alpha)s^2 + 10s + (5 + 20\alpha) = 0$ . What is the maximum value of  $\alpha$  can be assumed before the system becomes unstable? When  $\alpha$  is equal to the maximum value, the system oscillates. Determine the angular frequency of oscillation.
5. (16%) A unity feedback control system has the open-loop transfer function  $25/s(s+2)$ . Please derive and determine the resonant frequency  $\omega_p$ , peak resonant  $M_p$ , and bandwidth  $\omega_{BW}$ .
6. (20%) Please prove the following theorem for  $n = 4$ .  
The pair  $(A - \mathbf{b}\mathbf{k}, \mathbf{b})$ , for any  $1 \times n$  real constant vector  $\mathbf{k}$ , is controllable if and only if  $(A, \mathbf{b})$  is controllable.