本試題是否可以使用計算機：　□可使用　，　☑不可使用　　（請命題老師勾選）

(10 %)　1. Solve the following initial value problems:
$$y' = 2e^x y^3, \ y(0) = 0.5$$

(10 %)　2. Solve the following initial value problems:
$$y'' - 4y' + 4y = 0, \ y(0) = 0, \ y'(0) = -3$$

(15 %)　3. Solve the following initial value problems:
$$y''' - y'' - 4y' + 4y = 6e^{-x}$$

(15 %)　4. Let $f(n) = \mathbf{v}^T \mathbf{A}^n \mathbf{v}$, where $(\cdot)^T$ denotes transpose, n is a positive integer,

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}, \text{ and } \mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}. \text{ Please evaluate } \lim_{n \to \infty} \frac{\log_2 f(n)}{n}.$$

(20 %)　5. Find solutions $y(r, \theta)$ of the following boundary value problem for
$0 \le \theta \le \pi$ and $1 < r < \infty$ by separation of variables:

$$r \frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} y(r, \theta) \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta}\left(\sin \theta \frac{\partial}{\partial \theta} y(r, \theta) \right) = 0,$$
$$y(1, \theta) = \cos\theta, \quad 0 \le \theta \le \pi.$$

(15 %)　6. Expand $1/\sin z$ about $z = 0$ in $\pi < |z| < 2\pi$

(15 %)　7. Using the residue theorem to evaluate
$$\int_0^\pi \frac{\cos\theta}{\cos\theta - \cos\alpha} d\theta, \quad \alpha \text{ is a constant}$$