

1. For  $p$  a prime determine all elements  $a \in Z_p$  where  $a^2 = a$ . (10%)
2. Find the number of  $n$ -digit words generated from the alphabet  $\{0, 1, 2, 3, 4\}$  in each of which the total number of 0's and 1's is even. (15%)
3. Apply the state minimization process to the following machine. (15%)

	Next state		Output	
	0	1	0	1
$S_1$	$S_6$	$S_3$	0	0
$S_2$	$S_3$	$S_1$	0	0
$S_3$	$S_2$	$S_4$	0	0
$S_4$	$S_7$	$S_4$	0	0
$S_5$	$S_6$	$S_7$	0	0
$S_6$	$S_5$	$S_2$	1	0
$S_7$	$S_4$	$S_1$	0	0

4. On the first day of a new year, Joseph deposits \$1000 in an account that pays 6% interest compounded monthly. At the beginning of each month he added \$200 to his account. If he continues to do this for next four years (so that he makes 47 additional deposits of \$200), how much will his account be worth exactly four years after he opened it? (15%)
5. In how many different ways can we use two different colors to paint the faces of a cube. (15%)
6. Let  $f, g: Z^+ \rightarrow R$  where  $f(n) = n^2 + n$  and  $g(n) = (1/2)n^3$ . Please prove that  $f \in O(g)$  but  $g \notin O(f)$ . (15%)
7. Please find a 3 clock cycles scheduled data flow graph (the one like a state diagram) for the following computations and derive the minimum number of registers used in the graph using 2 adders and 1 multiplier. Assume both of the adder and the multiplier have one clock cycle delay. (Hint: using the graph coloring approach) (15%)

$$r = g + h + i$$

$$s = g + c + h * c$$