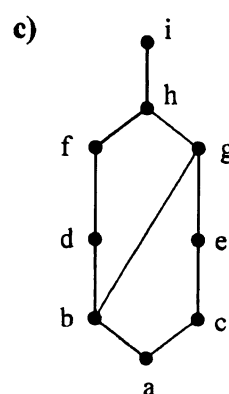
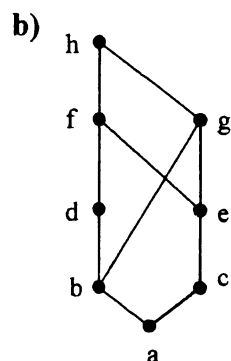
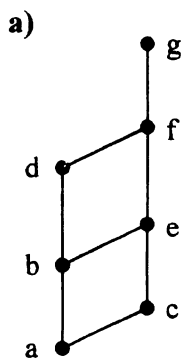


※ 考生請注意：本試題 可 不可 使用計算機

1. Let G be a loop-free undirected graph on n (≥ 3) vertices. If G has only one vertex of even degree, how many vertices in \bar{G} have even degree? (15%)
2. Let k, m be fixed integers. Find all values for k, m for which $(\mathbb{Z}, \oplus, \odot)$ is a ring under the binary operations $x \oplus y = x + y - k$, $x \odot y = x + y - mxy$, where $x, y \in \mathbb{Z}$. (15%)
3. In how many ways can the seven (identical) horses on a carousel be painted with black, brown and white paint in such a way that there are three black, two brown, and two white horses? (15%)
4. For $n \geq 1$, the n th triangular number t_n is defined by $t_n = 1 + 2 + \dots + n = n(n+1)/2$. Find and solve a recurrence relation for s_n , $n \geq 1$, where $s_n = t_1 + t_1 + \dots + t_n$, the sum of the first n triangular numbers. (15%)
5. Please derive a minimum state diagram of a *mealy*-style clocked sequential circuit with one input A and one output Z . When the circuit is turned ON, the output assumes the value of the first input signal. The output Z will then change value only if three consecutive input signals have the same *opposite* value of output Z . (15%)
6. Determine whether the posets with following Hasse diagrams are lattices and give your reasons. (15%)



7. Find the generating function for the following sequence (For example, in the case of sequence 0, 1, 3, 9, 27, ..., the answer required is $x/(1-3x)$, not $\sum_{i=0}^{\infty} 3^i x^{i+1}$ or simply $0 + x + 3x^2 + 9x^3 + \dots$):
 $0, 0, 1, a, a^2, a^3, \dots, a \neq 0$. (10%)