※ 考生請注意：本試題 □可 ☑不可 使用計算機

Problem 1. (15 pt)

// The recursive algorithm finds the largest and smallest elements in a sequence.

Input: $s_i, \ldots, s_j$, i and j

Outpt: *large* (the **largest** element in the sequence), small (the **smallest** element in the sequence)

```
1.  large_small (s, i,  j, large, small)
2.      if (i == j)  {
3.          large = s_i
4.          small = s_i
5.          return
6.      }
7.      m = ⌊ (i + j ) / 2 ⌋
8.      large_small (s, i, m, large_left, small_left)
9.      large_small (s, m +1, j, large_right, small_right)
10.     if (large_left > large_right)
11.         large = large_left
12.     else
13.         large = large_right
14      if (small_left > small_right)
15          small = small_right
16      else
17          small = small_left
18  }
```

Let b_n be the number of comparisons (lines 10 and 14) required for an input of size n. Please answer the following questions.

a. (5pt) Establish the recurrence relation of b_n.

b. (4pt) Find b_1 and b_2.

c. (6 pt) Solve the recurrence relation in case n is a power of 2 to obtain

$$b_n = 2n - 2, \qquad n = 1, 2, 4, \ldots$$

Problem 2. (10 pt)

Prove

$$n(1+x)^{n-1} = \sum_{k=1}^{n} C(n,k) k x^{k-1}$$

Problem 3. (10 pt) Figure 1 gives a graph with six nodes and thirteen edges. Each edge is associated with a weight. Please answer the questions in the following:

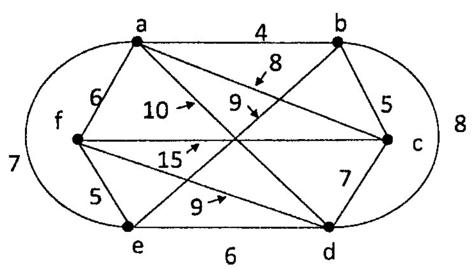

Figure 1. A graph.

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a. (2 **pt**) Decide whether the graph has an Euler cycle. If the graph has an Euler cycle, exhibit one; otherwise, briefly explain why it does not.

b. (3 **pt**) Determine whether the graph is planar. If the graph is planar, redraw it so that no edges cross; otherwise, show why it is not planar.

c.(5 **pt**) Find a minimal spanning tree using Prim's algorithm (suppose the source node is a).

Problem 4. (15 **pt**) Given a transport network, the source is ***a*** and the sink is **z**.

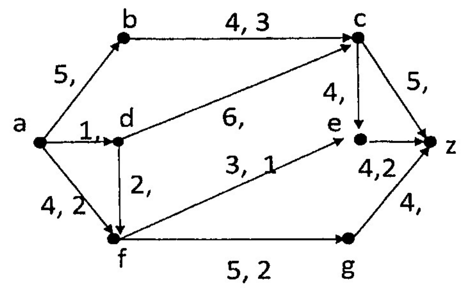

Figure 2. Edges are labeled *x*, *y* to indicate capacity *x* and flow *y*

a. (3 pt) Fill in the missing edge flows so that the result is a flow in the network.

b. (10 pt) Find the maximum flow and show the resulting edge flows.

c. (2 pt) Find the minimum cut cut(P, P') by showing the nodes in P.

Problem 5. (10 pt) A=B={1,2,3}; R={(1,1), (1,2), (2,3), (3,1)}; S={(2,1), (3,1), (3,2), (3,3)}. Let R and S be relations from a set A to a set B. Compute

(a) (5%) $\overline{R}$, (b) (5%) S^{-1}

Problem 6. (10 pt) A=ℝ and ≤denotes the usual partial order B={x | x is a real number and $5 \le x < 6$}. Find, if it exists,

(a) (5%) all lower bounds of B; (b) (5%) the least upper bound of B.

Problem 7. (10 pt) Using Karnaugh-map to simplify the Boolean function F=A'B'C'+B'CD'+A'BC'D'+AB'C'

Problem 8. (10 pt) Simplify the following Boolean function into product of sums form F(A,B,C,D)= Σ(0,1,2,5,8,9,10)

Problem 9. (10 pt) Consider the encoding function f defined below. How many errors will f detect?

f(000)=00000000
f(001)=10111000
f(010)=00101101
f(011)=10010101
f(100)=10100100
f(101)=10001001
f(110)=00011100
f(111)=00110001