系所組別：製造資訊與系統研究所丙組

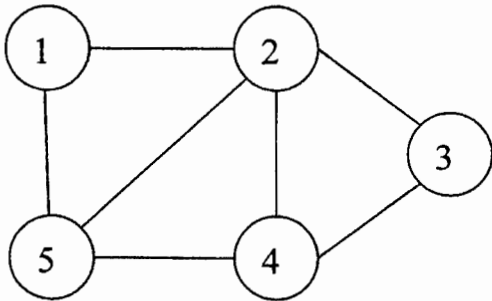考試科目：程式設計　　　　　　　　　　　　　　　　考試日期：0222，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一、Data Structures (50%)

1. (10%) Represent the following graph by adjacency-list and adjacency-matrix.

2. (10%) A max-heap is a heap such that for every node other than the root, the value of a node is at most the value of its parent.

    Is the array with value 〈23,17,14,6,13,10,1,5,7,12〉 a max-heap?.

3. (20%) For the set of {1,4,5,10,16,17,21} of keys, draw binary search trees of heights 2, 3, 4, 5, and 6.

4. (10%) The incident matrix of a directed graph $G = (V, E)$ with no self-loops is a

    $|V| \times |E|$ matrix $B = (b_{ij})$ such that

$$b_{ij} = \begin{cases} -1 & \text{if edge } j \text{ leaves vertex } i, \\ 1 & \text{if edge } j \text{ enters vertex } i, \\ 0 & \text{otherwise.} \end{cases}$$

Describe what the entries of the matrix product BB^T represent, where B^T is the transpose of B.

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二、Algorithms (50%)

5. (10%) Solving the recurrence $T(n) = T(n-1) + \dfrac{1}{n}$ using Θ notation.

6. (10%) Describe a $\Theta(n \log_2 n)$ -time algorithm that, given a set S of n integers and another integer x, determines whether or not there exist two elements in S whose sum is exactly x.

7. (10%) Describe a linear time algorithm to find strongly connected components in a directed graph.

8. (10%) Give a lower bound of any comparison sort algorithm to sort n numbers.

9. (10%) (a) (5%) Determine which one of the **0-1 knapsack problem** and the **fractional knapsack problem** cannot be solved using the greedy strategy? (b) (5%) Give an example to explain that.