※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一、Data Structures (50%)

1. (10%) What are the minimum and maximum numbers of elements in a heap of height h?

2. (10%) A min-heap is a heap such that for every node other than the root, the value of a node is at least the value of its parent.
   Is an array that is in sorted order a min-heap?.

3. (15%) Let (u,v) be a minimum-weight edge in a connected graph G. Show that (u,v) belongs to some minimum spanning tree of G.

4. (15%) Show all legal B-trees of minimum degree 2 that represents {1,2,3,4,5}.

二、Algorithms (50%)

5. (10%) Solving the recurrence $T(n) = T(\dfrac{9n}{10}) + n$ using Θ notation. Assume that $T(n)$ is constant for sufficiently small n.

6. (10%) The matrix-chain multiplication problem can be stated as follows: Given a chain $\langle A_1, A_2, \ldots, A_n \rangle$ of n matrices, where for $i=1,2,\ldots,n$, matrix A_i has dimension $p_{i-1} \times p_i$, fully parenthesize the product $A_1 A_2 \cdots A_n$ in a way that minimizes the number of scalar multiplications. Suppose that you have 6 matrices: A_1 has dimension 30×35, A_2 has dimension 35×18, A_3 has dimension 18×5, A_4 has dimension 5×10, A_5 has dimension 10×25, A_6 has dimension 25×30. Please calculate the minimum number of scalar multiplications.

7. (10%) Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$?

8. (10%) Prove or disprove: The single-source shortest paths problem can be solved in linear time in directed acyclic graphs.

9. (10%) We are given a directed graph $G=(V, E)$ on which each edge $(u,v) \in E$ has an associated value $r(u,v)$, which is a real number in the range $0 \le r(u,v) \le 1$ that represents the reliability of a communication channel from vertex u to vertex v. We interpret $r(u,v)$ as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent. Given an efficient algorithm to find the most reliability path between two given vertices. .