

1. 試用拉氏轉換法 (Laplace Transform) 解初始值問題。

$$xy'' + 2y' + (2-x)y = 2e^x$$

$$y(0) = 0.$$

(10%)

2. (a). 求逆拉氏轉換 (Inverse Laplace Transform)

$$\mathcal{L}^{-1} \left[\frac{1}{s^2} (1 - \exp(-s))^2 \right] = ?$$

(6%)

(b) 函數 $f(t)$ 可做 Laplace Transform 的條件是什麼? (2%)

$$(c) \mathcal{L}[f'(t)] = s\mathcal{L}[f(t)] - f(0^+)$$

能成立時, $f(t)$, $f'(t)$ 需具備什麼條件? (2%)

3. 試解偏微分方程式

$$\frac{\partial^2 \theta}{\partial t^2} = c^2 \frac{\partial^2 \theta}{\partial x^2} \quad 0 < x < L, \quad t > 0. \quad c^2 \text{ 為常數.}$$

$$\text{其邊界條件: } \theta(0, t) = 0 \quad \theta(L, t) = 0 \quad t > 0$$

$$\text{起始條件: } \theta(x, 0) = 0 \quad \left. \frac{\partial \theta}{\partial t} \right|_{t=0} = \omega_0 x / L \quad 0 < x < L$$

(10%)

4. 試求出矩陣 $[A]$ 之特徵值及其對應之特徵向量

$$[A] = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

(12%)

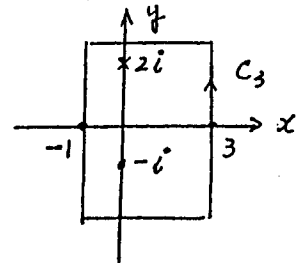
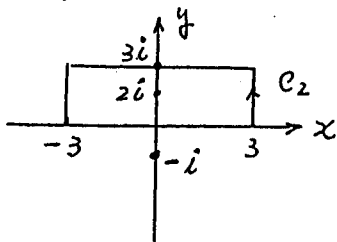
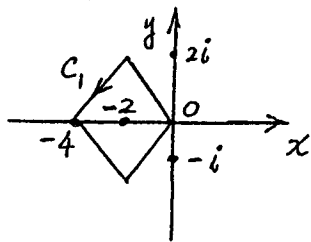
5. 試將函數 $F(x_1, x_2) = 3x_1^2 + 3x_2^2 + 2x_1x_2$ 轉換成

標準式 (Canonical form).

(16%)

6. 試求出 $I_j = \oint_{C_j} \frac{\sin(z)}{z^2 - iz + 2} dz$ 之積分值

其中 C_j 為 C_1 , C_2 , 和 C_3 並分別對應下列的圖形. (12%)



7. Solve $x y'' + 2 y' = 4 x^3$ (10%)

8. Show that the vector field

$$\vec{F} = (y^2 \cos x + z^3) \vec{i} + (2y \sin x - 4) \vec{j} \\ + (3xz^2 + 2) \vec{k}$$

is conservative, and deduce the corresponding
scalar potential Φ (20%)