

6. Applying the Laplace transform method to solve the differential equation: (10%)

$$y'' + 2ty' - 4y = 0, \quad \text{with } y(0) = y'(0) = 0$$

7. Let $F = -i + xyzj - y^2k$, and let C be given by $x = t$, $y = |t|$, $z = 1$, $t = -1$ to 1 , compute line integral $\int_C F = ?$ (10%)

8. Evaluate $\int_C \frac{e^z}{z^2(z-i)}$, where C encloses 0 and i . (10%)

9. Solve the given partial differential equation: (10%)

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, \quad t > 0, \quad a^2 \text{ is constant.}$$

$$\text{I.C.: } u(x, 0) = 2x, \quad 0 < x < L.$$

$$\text{B.C.: } \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(L, t) = 0, \quad t > 0.$$

1. Compute $\int_0^1 \frac{1}{x^k} dx = ?$, k is constant. (5%)
2. Find the maximum and minimum values of $6x+3y+2z-5$ on the surface $4x^2+2y^2+z^2=70$. (10%)
3. Compute the volume of the solid bounded by the circular cylinders $x^2+y^2=1$ and $x^2+z^2=1$. (10%)
4. Determine the general solution of the following differential equations:
 - (a). $xy''+2y' = 4x^3$ (10%)
 - (b). $y''+y = \sec x$, $0 < x < \pi/2$ (10%)
5. A system of homogeneous linear equations with constant coefficients.

$$\mathbf{x}' = \mathbf{A}\mathbf{x} \quad (1)$$

Where \mathbf{A} is a matrix,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

- (a). Find the eigenvalue of $[\mathbf{A}]$. (5%)
- (b). Find the eigenvector of $[\mathbf{A}]$. (5%)
- (c). Give the general solution of Eq. (1). (5%)