線性代數(48%)

(8). For any Vectors u, v∈C^n and scalar z∈C, prove $u\cdot v=\overline{v\cdot u}$. (8%)

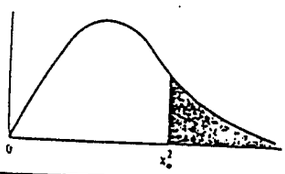
(9). Determine whether the vectors u, v and w are dependent or independent. (8%)
u=(1, -2, 3, 1), v=(3, 2, 1, -2), w=(1, 6, -5, -4)

(10). Let T: $R^3\to R^3$ be the linear mapping defined by T(x, y, z)=(x+2y-z, y+z, x+y-2z)
Find a basis and dimension of the (i)image U of T, (ii)Kernel W of T. (16%)

(11). Let $A=\begin{pmatrix}1 & 4\\ 2 & 3\end{pmatrix}$. Find all eigenvalues of A and the corresponding eigenvectors. (8%)

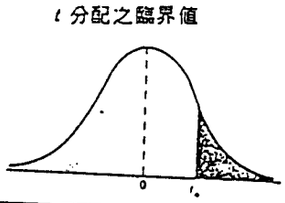
(12). Let $A=\begin{pmatrix}1 & 2 & 3\\ 0 & 2 & 3\\ 0 & 0 & 3\end{pmatrix}$. Is A similar to a diagonal matrix? If so, find one such matrix. (8%)

卡方分配之臨界值



v	α 0.995	0.99	0.975	0.95	0.05	0.025	0.01	0.005
1	0.0^4393	0.0^3157	0.0^3982	0.0^2393	3.841	5.024	6.635	7.879
2	0.0100	0.0201	0.0506	0.103	5.991	7.378	9.210	10.597
3	0.0717	0.115	0.216	0.352	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	11.070	12.832	15.086	16.750
6	0.676	0.872	1.237	1.635	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672

t 分配之臨界值



v	α 0.10	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
inf	1.282	1.645	1.960	2.326	2.576

機率與統計(52%)

(1). 一不透明箱中含2黑球及2白球，任取兩球，試求各種情況發生之機率。(5%)

(2). 若Y為一連續型隨機變數，且機率函數為

$$f(y) = \frac{3}{8}y^2 \quad 0 \le y \le 2$$
$$= 0 \quad \text{elsewhere}$$

(a). 試求F(y)。 (5%)
(b). 試繪F(y)之機率分配圖。 (3%)

(3). 設Gamma隨機變數之型式如下：

$f(y) = \frac{y^{\alpha-1} \cdot e^{-y/\beta}}{\Gamma(\alpha)\beta^{\alpha}}$，其中$\alpha$、$\beta > 0$，$0 \le y \le \infty$，且$\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1} \cdot e^{-y} dy$

試求其期望值。 (8%)

(4). 試證$Var(Y) = E(Y^2) - (E(Y))^2$。
隨機變數Y之變異數之定義為$Var(Y) = E(Y-\mu)^2$。(7%)

(5). 某一射出成型零件廠聲稱其每日產品之不良數為13。今任意自該廠抽取49天之不良資料而得平均不良數為14，且標準差為2，試問該廠所聲稱是否屬實？
若$\alpha = 5\%$ (8%)

(6). 某廠稱其所製造之產品尺寸變異大小不超過2cm。某客戶為了求證該廠所言是否屬實，任取8個產品進行檢定，若其變異數大小為2cm，試問在$\alpha = 5\%$下，該公司所言是否屬實。(設母體為常態分配) (8%)

(7). 某公司產品過去五年之銷售額如下表所示，試估計未來五年之趨勢。 (8%)

(單位：仟萬元)

年	x	y	x^2	x y	$\hat{y}$
1994	2	16	4	32	16.4
1993	1	13	1	13	12.9
1992	0	10	0	0	9.4
1991	-1	6	1	-6	5.9
1990	-2	2	4	-4	2.4
總和	0	47	10	35	