5. Solve for $y(t)$ from the simultaneous equations

$$y' + 2y + 6\int_0^t z\,dt = -2u(t)$$

$$y' + z' + z = 0$$

if $y_0 = -5$ and $z_0 = 6$.

[Hint: Use Laplace transform] (12%)

6. (a) Show that the vectors

$$V_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix},\quad V_2 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix},\quad V_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix},\quad \text{and } V_4 = \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix}$$

are linearly dependent. (6%)

(b) Show that the vectors V_1, V_2, and V_3 shown above are linearly independent. (6%)

7. What is the directional derivative of the function $\phi(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the vector $i + 2j + 2k$? (13%)

8. If $p(x) = x^4 - 4x^3 + 6x^2 - x - 3$

and $A = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$

find $p(A)$. (13%)

1. Determine the solution to the differential equation (10%)

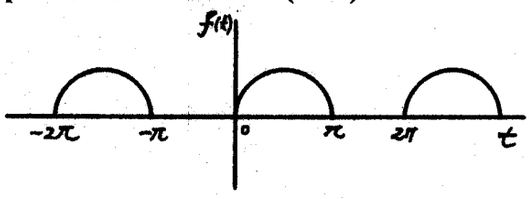
$$x^2\frac{d^2u}{dx^2} - 5x\frac{du}{dx} + 8u = 0$$ with I.C. u (1) = 2, $u'(1) = 8$.

2. Evaluate the real integral (10%)

$$\int_0^\infty \frac{dx}{(1+x^2)} = ?$$

3. Find the Fourier series for the periodic extension of (10%)

$$f(t) = \begin{cases} \sin t, 0 \le t \le \pi \\ 0, \pi \le t \le 2\pi. \end{cases}$$



4. A 2 m long, laterally insulated stainless steel rod, has heat generation occurring within the rod at constant rate of 10000 W/m^3. The left end is maintained at 0 ℃ and the right end is insulated. Use the specific heat C = 500 J/kg.℃, ρ = 8000 kg/m^3, and k = 4.0 x 10^{-6} m^2/s.

(a). Give the governing equation. (5%)
(b). If the I.C. is

$$T(x,0) = f(x) \begin{cases} = 100x, & 0 < x < 1. \\ = 200 - 100x, & 1 < x < 2. \end{cases}$$

Find the solution for T(x,t). (15%)

(背面仍有題目,請繼續作答)