一. C is the line section between $(1, 1, 1)$ and $(-4, 2, 5)$;

F is $F = -5yz\, dy - z^3 dz$. Find $\int_C F = ?$ (15%)

二. Give the particular solution of $y(x)$ in the following ef.:

$$x^2 y'' + 3xy' + y = 4/x$$ (10%)

三. Use Laplace method to solve the following initial value problem:

$$y'' + 4y = f(t) \quad ; \quad f(t) = \begin{cases} 0 & , \ 0 \le t < 4 \\ 3 & , \ t \ge 4 \end{cases}$$

I.C. $\begin{cases} y(0) = 1, \\ y'(0) = 0, \end{cases}$ (15%)

四. Solve the following partial differential equation: (10%)

$$\frac{\partial^2 y}{\partial t^2} = 9 \frac{\partial^2 y}{\partial x^2} + x^2 \quad ; \quad 0 < x < 4, \ t > 0$$

with

B.C. $y(0, t) = y(4, t) = 0$ $(t > 0)$

I.C. $y(x, 0) = \frac{\partial y}{\partial t}(x, 0) = 0$ $(0 < x < 4)$.

（背面仍有題目,請繼續作答）

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5. What is the solution of the differential equation
(12%)
$$y'' + 2y' + y = te^{-t}$$
for which $y_0 = 1$ and $y_0' = -2$?

6. Find a sequence of elementary row transformations which
(13%) will reduce the matrix

$$P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

to I_3; determine the matrices $M_1, M_2, \cdots$, corresponding to these transformations; and use these results to compute the inverse of P.

7. Solve the equation
(13%)
$$X^2 - 4X + 4I = \begin{bmatrix} 4 & 3 \\ 5 & 6 \end{bmatrix}.$$

8. If A, B, and C are three vectors which are not parallel
(12%) to the same plane, show that any vector V can be expressed as a linear combination of A, B, C as:

$$V = \frac{[VBC]}{[ABC]} A + \frac{[AVC]}{[ABC]} B + \frac{[ABV]}{[ABC]} C.$$

$$\left\{ \begin{array}{l} \text{Hint}: [ABC] = A \cdot B \times C = C \cdot A \times B = A \times B \cdot C \\ \text{is a } \underline{\text{scalar triple product}}. \end{array} \right\}$$