壹.

Differential Equations

1. Solve the differential equation $dy/dx = -4xy^2$ (7%)
   and then solve the initial-value problem
   $dy/dx = -4xy^2$, $y(0) = 1$.

2. Find a curve in the xy-plane that passes through (0, 3) (10%)
   and whose tangent line at a point (x, y) has slope $2x/y^2$.

3. **Euler's Method**: (8%)
   To approximate the solution of the initial-value problem

   $$y' = f(x, y),\ y(x_0) = y_0$$

   proceed as follows:

   Step 1. Choose a nonzero number h to serve as an **increment** or **step size** along the x-axis, and let
   $$x_1 = x_0 + h,\quad x_2 = x_1 + h,\quad x_3 = x_2 + h, \ldots$$

   Step 2. Compute successively
   $$y_1 = y_0 + f(x_0, y_0)h$$
   $$y_2 = y_1 + f(x_1, y_1)h$$
   $$y_3 = y_2 + f(x_2, y_2)h$$
   $$\ldots$$
   $$y_{n+1} = y_n + f(x_n, y_n)h$$

   The number y_1, y, y_3... in these equations are the approximations of $y(x_1), y(x_2), y(x_3), \ldots$

   Use Euler's Method with a step size of 0.1 to make a table of approximate values of the solution of the initial-value problem
   $$y' = y - x,\quad y(0) = 2$$
   over the interval $0 <= x <= 0.5$.

(背面仍有題目,請繼續作答)

貳、 The general linear second order partial differential equation (P.D.E.) in three variables (one dependent and two independents) is,

$$a(x,y)\frac{\partial^2 u}{\partial x^2}+2b(x,y)\frac{\partial^2 u}{\partial x\partial y}+c(x,y)\frac{\partial^2 u}{\partial y^2}+d(x,y)\frac{\partial u}{\partial x}$$
$$+e(x,y)\frac{\partial u}{\partial y}+f(x,y)u(x,y)+g(x,y)=0 \tag{1}$$

Most of the equations we encounter will be of this form. Please explain (briefly) the following questions based on the above equation.

(a) What is homogeneous P.D.E.? (2%)

(b) What is well posed problems involving a P.D.E.? (2%)

(c) What is hyperbolic P.D.E.? (2%)

(d) What is elliptic P.D.E.? (2%)

(e) What is parabolic P.D.E.? (2%)

(f) Give an engineering example involving the hyperbolic P.D.E. (5%)

(g) Give an engineering example involving the elliptic P.D.E. (5%)

(h) Give an engineering example involving the parabolic P.D.E. (5%)

備註:(f)、(g)、(h)的答案只須包含 1.統制方程 2.邊界或初始條件 3.輔助說明圖(不須寫出求解計算過程)。

參、(a) Given the initial conditions $y=0$, $\frac{dy}{dt}=0$, and $\frac{d^2y}{dt^2}=0$,
[15%] write the Laplace transform of the following differential equation:

$$2\frac{d^2y}{dt^2}+10\frac{dy}{dt}+12.5y=16.$$

Solve the differential equation by using the inverse transform method.

(b) Use the initial value theorem and the final value
[10%] theorem to determine the values of y at time $=0$ and $t=\infty$, respectively, for the following:

(i) $y(s)=\dfrac{13}{s^2+18}$

(ii) $y(s)=\dfrac{s+2}{s(s^2+9s+16)}$

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1 Let A and B be $n \times n$ orthogonal matrices. A' and B' are the transpose matrices for A and B, respectively. "det A" is the determinant of matrix A. Prove:
a) A' and A^{-1} are orthogonal. (10%)
b) AB is also orthogonal. (5%)

2 In V^4 (4-dimensional space), let $u = (3,2,1,0)$, $v = (1,0,1,2)$, $w = (5,4,1,-2)$. Are u, v, w linearly independent? (10%)