1. Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and $f(x) = 3x^2 - 2x + 4$.

   a) Find eigenvalues and associated eigenvectors for A. (10%)
   b) Find eigenvalues for $f(A)$. (10%)

2. Let X have the following cdf (cumulative distribution function)

$$F(x) = \begin{cases} 1 - e^{-x^2}, x \ge 0 \\ 0, \ \text{elsewhere} \end{cases}$$

   Define two new random variables Y and Z by

$$Y = X^2 + 1$$

   and

$$Z = \begin{cases} 1 & \text{if } 0 \le X < 1 \\ 2 & \text{if } 1 \le X < \sqrt{2} \\ 3 & \text{if } X \ge \sqrt{2} \end{cases}$$

   a) Find the cdf of Y. (10%)
   b) Find the cdf of Z. (10%)

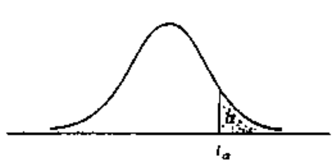
3. Define precisely the Central Limit Theorem for sampling means of a sampling distribution. Assume that a random sample of size n is drawn from a population with mean μ and variance σ^2. (15%)

4. Find the slope of the curve defined below. (15%)

$$y = \frac{L}{1 + ae^{-bx}}, \text{ where } L, a, \text{ and } b \text{ are constants.}$$

5. Use the following data to construct a 95% confidence interval estimate of the true mean.
   (253, 261, 258, 255, and 256) You may use the table provided below. (15%)

6. Let $z = x^3 - 3x^2y$, where x and y are functions of t such that for $t = 5$, $x = 7$, and $y = 2$, $dx/dt = 3$, and $dy/dt = -1$. Find dz/dt for $t = 3$. (15%)



Degrees of Freedom	$t_{0.100}$	$t_{0.050}$	$t_{0.025}$	$t_{0.010}$	$t_{0.005}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.695	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055