

1. (a) Construct the block diagram that combines the following [10%] set of equations expressed in the s -notation:

$$w = x - y$$

$$v = w - z$$

$$z(s+6) = v(s+2)$$

$$y(s^2 + 6s + 8) = z$$

Let x be the input to the system and y be the output.

(b) Reduce the block diagram determined in (a) into a [10%] single transfer function with $x = \text{input}$ and $y = \text{output}$.

2. Consider the unity-feedback control system whose open-loop [20%] transfer function is

$$G(s) = \frac{100}{s(0.1s + 1)}$$

Determine the steady-state error when the input is

$$r(t) = 1 + t + at^2, \quad a \geq 0.$$

3. Consider the following open-loop transfer function:

$$G(s)H(s) = \frac{K}{s(s+1)(2s+1)}$$

Determine the critical value of K for stability

by use of (a) Routh's stability criterion [10%]

(b) Nyquist stability criterion [10%].

4. Assume that the block diagram in Fig. 1 represents the model for some industrial process (time units are seconds):
- [2%] (a) Determine the open-loop transfer function for this system.
- [2%] (b) Determine the closed-loop transfer function for this system.
- [10%] (c) Plot the root locus for this system.
- [6%] (d) The specifications on this system are that the maximum value of the time constant is to be $\tau = 1.5$ s and that there is to be no oscillation in the response. Is this performance possible to achieve in this system and, if so, what is the corresponding value of the gain factor K ?

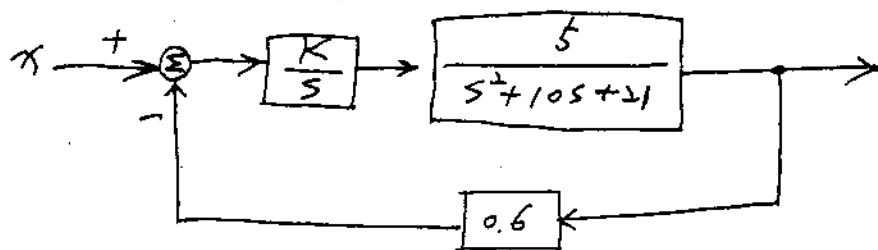


Figure 1.

5. (a) Obtain the state-transition matrix $\Phi(t)$ of the following system.

[10%]
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Obtain also the inverse of the state-transition matrix, $\Phi^{-1}(t)$.

- (b) Obtain the time response of the following system:

[10%]
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

where $u(t)$ is the unit-step function occurring at $t=0$.

{ Noting that the A matrices of (a) and (b) are the same and equal to $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ }