壹.

(a) Solve the initial value problem (10%)

$$y'' + 2y' + 3y = 0; \quad y(1) = 2, \quad y'(1) = -3.$$

(b) Find the general solution of (15%)

$$y^{(3)} + 2y'' - y' = 4e^{x} - 3\cos(2x).$$

貳.

(a) Find Laplace:

$$\mathcal{L}[e^{-3t}f(t)] = ? \quad f(t) = \begin{cases} 0, & t < 8 \\ t^2 - 4, & t \ge 8. \end{cases}$$

(15%)

(b) Find Inverse Laplace:

$$\mathcal{L}^{-1}\left(\frac{2s^2 - s}{(s^2+4)^2}\right) = ?$$

(10%)

(背面仍有題目,請繼續作答)

參.

Consider the heat equation for a bar with no heat flow across the ends: (25%)

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \qquad (0 < x < L,\ t > 0)$$

$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(L,t) = 0 \qquad (t > 0)$$

$$u(x, 0) = x(L-x) \qquad (0 < x < L)$$

Solve the temperature distribution u(x, t).

肆. 行列式與矩陣 (25%)

1 Solve the linear system by Gauss-Jordan reduction (5%)

$$x + 2y + 3z = 9$$
$$2x - y + z = 8$$
$$3x - z = 3$$

2 Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix}$$ if it exists. (5%)

3 Prove that if u and v are vectors in R^n, then

$\|u + v\| \le \|u\| + \|v\|$. (10%)

4 Let $L: R^3 \to R^2$ be a linear transformation for which we know that

$L(1,0,0) = (2,-1)$, $L(0,1,0) = (3,1)$, and $L(0,0,1) = (-1,2)$.

Find $L(-3,4,2)$. (5%)