1 Find an equation of the plane passing through the points $P_1(2,-2,1)$, $P_2(-1,0,3)$ and $P_3(5,-3,4)$ (15%)

2 Two methods, A and B, are available for teaching a certain industrial skill. The failure rate is 20% for A and 10% for B. However, B is more expensive and hence is used only 30% of the time (A is used for other 70%). A worker is taught the skill by one of the methods but fails to learn it correctly. What is the probability that he was taught by method A? (10%)

3 Using the definition of conditional probability, show that

$P(ABC) = P(A)P(B|A)P(C|AB)$. (10%)

4 Prove that for any random variable X and constants a and b,

(1) $E(aX+b) = aE(X) + b$ (5%)

(2) $V(aX+b) = a^2V(X)$ (10%)

5 The daily production of electrical motors at a certain factory averaged 120 with a standard deviation of 10.

(1) What fraction of days will have a production level between 100 and 140? (5%)

(2) Find the shortest interval certain to certain at least 90% of the daily production levels. (10%)

Hint: Tchebysheff's Theorem, Let X be a random variable with mean μ and variance σ^2. Then for any positive k, $P(|X-\mu|<k\sigma)\geq 1-1/k^2$

6 Show that the moment-generating function for the binomial random variable is given by

$M(t) = [pe^t + (1-p)]^n$

(背面仍有題目,請繼續作答)

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Us this result to derive the mean and variance for the binomial distribution. (15%)

Hint: $\sum_{y=0}^{n}\binom{n}{y}a^{y}b^{n-y}=(a+b)^{n}$

7 Suppose X_1, X_2, X_3 denotes a random sample from the exponential distribution with density function

$$f(x)=\begin{cases}\frac{1}{\theta}e^{-x/\theta}, x>0\\ 0, otherwise\end{cases}$$

Consider the following four estimators of θ:

$$\hat{\theta}_1=X_1$$

$$\hat{\theta}_2=\frac{X_1+X_2}{2}$$

$$\hat{\theta}_3=\frac{X_1+2X_2}{3}$$

$$\hat{\theta}_4=\bar{X}$$

(1) which of the above estimators are unbiased for θ? (10%)

(2) Among the unbiased estimators of θ, which has the smallest variance? (10%)