壹、

(a). Find the general solution of (10%)

$$y^{(4)} + 11y^{(3)} + 36y'' + 16y' - 64y = -3e^{-4x} + 2\cos(2x)$$

Hint: The characteristic equation is

$$(r+4)^3(r-1) = 0$$

(b). Solve the follow equation (y(x)= ?) (10%)

$$x^2y'' + 5xy' - 2y = 0$$

貳、Consider a mass-spring system as in Fig.1 with a periodic driving force $A\sin(\omega t)$ and no damping. Assuming that the mass is initially at rest in the static equilibrium position, then the motion is governed by

$$my'' + ky = A\sin(\omega t),$$
$$y(0) = y'(0) = 0$$

(a). Using Laplace transformation method to solve y(t) =? (20%)

(b). Show the result in (a) with cases: (5%)

(1). $\omega \neq \omega_0$, and (2). $\omega = \omega_0$

(Here $\omega_0^{\ 2} = k/m$)

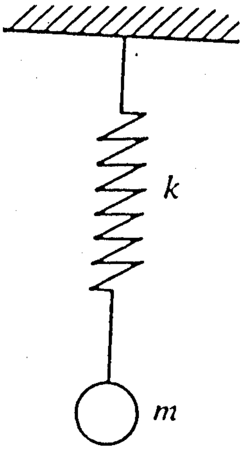


Fig. 1

(背面仍有題目,請繼續作答)

參、Consider the mass-spring system of Fig.2, in which y_1 and y_2 measure displacements of masses m_1 and m_2, respectively, from equilibrium positions. The spring constants are $k_1=5$ and $k_2=6$ as shown, and we chose $m_1=m_2=1$。Assume no damping and no external driving forces.

The motion is governed by:

$$y_1'' = -(k_1+k_2)y_1 + k_2 y_2,$$
$$y_2'' = +k_2 y_1 - k_2 y_2,$$

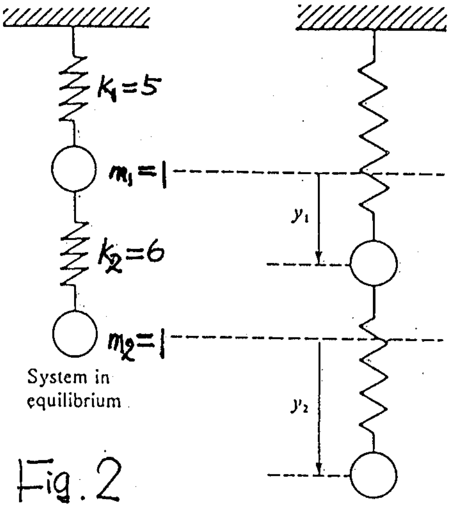


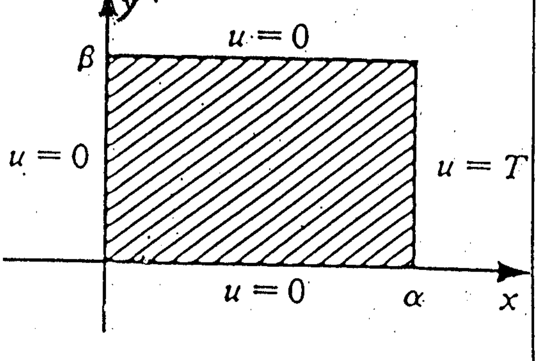
Fig.2

(a). Find the eigenvalue of the system in Fig.2. (10%)
(b). Find the eigenvector fo the system in Fig.2. (10%)
(c). Find the general solution of the system in Fig.2. (10%)

肆、Consider steady-state heat conduction in a flat plate having temperature values prescribed on the sides, as shown in Fig.3. The boundary value problem modeling this is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad (0 < x < \alpha,\ 0 < y < \beta),$$

$$u(x,0) = u(x,\beta) = 0 \qquad (0 < x < \alpha),$$
$$u(0,y) = 0 \qquad (0 < y < \beta),$$
$$u(\alpha,y) = T \qquad (0 < y < \beta).$$



Find the solution of $u(x, y) = ?$ (25%)

Fig.3