一、Find a general solution of the following equation (10%)

$$y'' - 5y' + 6y = -3\sin(2x).$$

二、Let $z = \cos(x, y)$, with x and y any numbers. Find the normal vector on the surface at any point (x, y, z). (10%)

三、Let $R = 3t^2\vec{i} - \sin(t)\vec{j} + 2t\vec{k}$, for $a \le t \le b$.
Where s is the arc length along the curve described by R.
Please find $dR/ds = ?$ (7%)

四、Let $F = -\vec{i} + xyz\vec{j} - y^2\vec{k}$, and (8%)
let C be given by $x = t, \quad y = |t|, \quad z = 1; \quad t: -1 \to 1$
Please find $\int_c F = ?$

五、Solve the following boundary value problem (15%)

$$\frac{\partial^2 y}{\partial t^2} = 9\frac{\partial^2 y^2}{\partial x^2} \qquad (0 < x < L,\ t > 0)$$

With B.C.

$$y(0, t) = y(L,\ t) = 0 \qquad (t > 0)$$
$$y(x, 0) = 0 \qquad (0 < x < L)$$
$$\frac{\partial y}{\partial t}(x,\ 0) = g(x) \qquad (0 < x < L)$$

(背面仍有題目,請繼續作答)

六. Use Laplace transformation method to solve for $x(t)$ from the simultaneous equations:

$$\begin{cases} \dot{x} + x + 3\int_0^t y\,dt = \cos t + 3\sin t \\ 2\dot{x} + 3\dot{y} + 6y = 0 \end{cases}$$

with $x(0) = -3$, $y(0) = 2$. (20%)

七. For $A = \begin{Vmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 3 & 1 & -2 \end{Vmatrix}$

find the determinant and inverse of A. (10%)

八. Find the local extrema of the function:

$$f(x_1, x_2, x_3) = 35 - 6x_1 + 2x_3 + x_1^2 - 2x_1x_2 + 2x_2^2 + 2x_2x_3 + 3x_3^2$$

and justify that these extrema are local maximum or local minimum. (20%)